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Beyond GPS: Vehicle Positioning Using Wheel Sensors, Motion Models and Sensor Fusion

An Evaluation of Dead Reckoning Methods and Sensor
Fusion for Heavy Vehicles

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Abstract

Accurate vehicle positioning is becoming increasingly important for modern transportation systems, especially with the recent focus on advanced driver assistance systems and autonomous driving. Currently Global Navigation Satellite System (GNSS) is the main source for vehicle positioning information. However, GNSS can suffer from reduced accuracy or complete signal loss due to signal blockage or multi-path errors, particularly in urban environments. In systems that rely on accurate positioning, such failures could pose a major safety risk to passengers, other road users and surrounding infrastructure. Therefore, this thesis investigates an alternative solution to estimate vehicle position. By utilizing data from wheel speed sensors and steering angle sensors together with motion models, vehicle trajectories are calculated and evaluated. The study also investigates how motion models can be combined with GNSS measurements using an extended Kalman filter (EKF) with the goal of improving heavy vehicle positioning. In total, five different motion models, three bicycle models of varying complexity, one rear axle model and one front axle model are evaluated using data collected at a test track. An RTK GNSS is used as the ground truth for the experiments.

The results show that motion model selection and implementation has a large impact on the dead reckoning performance. Simpler kinematic models perform better in driving conditions with lower velocity, acceleration and turning. The dynamic bicycle model does not perform as well in less demanding conditions but show good results when tested in more challenging driving conditions. The rear axle model performs accurately in experiments with constant velocity or heading, but becomes less accurate during wide turns and velocity changes. The front axle model performs worse than all other models regardless of the driving conditions. When compared to GNSS, the dead reckoning models achieve higher positioning accuracy for short periods of approximately 10 - 20 seconds, after which the trajectory starts to drift due to accumulated errors. The EKF sensor fusion approach produce a more accurate and robust trajectory estimate compared to standalone dead reckoning. Overall, the results suggest that wheel sensor-based dead reckoning can be combined with GNSS through sensor fusion to enhance heavy vehicle positioning accuracy, or serve as a short-term complementary positioning solution during GNSS outages.

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Populärvetenskaplig sammanfattning

Exakt och robust positionering av fordon är ett grundläggande krav för avancerad fordonsteknik, som i autonoma fordon, trafikplanering, och intelligenta transportsystem. Tillförlitlig positionering möjliggör funktioner som kollisionsundvikande, automatisk filhållning, bromsning och parkering. Detta bidrar till ökad trafiksäkerhet och kan också förbättra trafikledningssystem. Dessa system kan minska köer, förbättra effektiviteten i trafiken och därigenom spara tid och minska miljöpåverkan från transport. I takt med den snabba utvecklingen inom autonoma fordon har behovet av robust och noggrann positionsinformation ökat markant de senaste åren.

Sedan 1990-talet har Global Navigation Satellite System (GNSS), i vardagligt tal GPS, varit en av de viktigaste teknikerna för att bestämma fordons position. Genom att ta emot signaler från ett flertal satelliter så kan positionen av mottagaren bestämmas med en noggrannhet på ungefär 0-5 meter. Trots sina fördelar så finns det tillfällen när denna teknik inte fungerar som tänkt, till exempel när signalerna från satelliterna studsar eller blockeras av objekt i omgivningen, som höga byggnader, broar eller vid körning i tunnlar. Risken för avsiktliga GNSS-störningar har även ökat på senare år. Detta kan leda till försämrad eller helt utebliven positionering, som kan utgöra en risk om fordons system är beroende av korrekt positionsinformation för att fungera, till exempel i assisterade eller autonoma fordon.

Utöver GNSS finns det flera tekniker för att bestämma ett fordons position. En metod är att använda sig av hjulhastighetssensorer och styrvinkelsensorer för att beräkna hur långt och med vilken typ av sväng fordonet har förflyttat sig. En annan metod är att använda sensorer som med hjälp mäter fordonets acceleration och rotation och använder dessa för att estimerar fordonets rörelse. Genom att kontinuerligt beräkna fordonets rörelser med dessa sensorer kan fordons positionen uppskattas steg för steg, en metod som kallas för dödräkning. Teoretiskt sett kan denna typ av sensorer ge en perfekt positionering. I praktiken påverkas mätningarna av felkällor som leder till att estimeringen över tid bygger upp ett större och större fel. Ett exempel är när hjulen slirar, då kommer sensorerna registrera att fordonet rör sig längre än det faktiskt har. Detta kommer i längden leda till att positionsfelet gradvis ökar.

Denna uppsats har undersökt hur hjulhastighet- och hjulvinkelsensorer kan användas för att dödräkna positionen av tunga fordon. Vidare har det undersökts vilka matematiska modeller som ger bäst resultat under olika tester. Dessa modeller använder data från sensorerna för att beräkna hur fordonet rör sig över tid. Detta betyder att modellerna inte använder GNSS, utan endast förlitar sig på sensorerna. För att testa hur bra modellerna presterade användes en Real Time Kinematic (RTK) GNSS, vilket är en typ av GNSS som använder sig av en lokal basstation som ökar noggrannheten i mätningarna. Denna typ av GNSS har en noggrannhet på några centimeter, och därför kunde värden från RTKn användas som valideringsdata.

För att kombinera GNSS-data med mätningar från sensorerna användes ett Extended Kalman filter (EKF). Filtret fungerar genom att först göra en uppskattning av vart fordonet borde befinna sig baserat på tidigare position och en rörelsemodell. Sedan jämförs denna uppskattning med den nästkommande positionen från GNSS-mätningen. Baserat på osäkerheterna i både modell- och GNSS-mätningarna beräknar filtret en uppdaterad positionsuppskattning.

Totalt testades fem olika modeller, tre cykelmodeller och två axelmodeller. De tre cykelmodellerna var en kinematisk modell, en kinematisk modell som tar hänsyn till att hjulen glider och en dynamisk modell som tar hänsyn till krafter och mer komplexa parametrar. Axelmodellerna modellerade bakaxeln och framaxeln på fordonet.

Resultatet visade att valet av modell var avgörande för hur bra lastbilens position kunde uppskattas. Ingen av modellerna gav noggrann positionering i alla situationer, utan de presterade olika noggrant i olika tester. De enklare kinematiska cykelmodellerna presterade bra på långsammare körningar, medan de mer avancerade modellerna presterade bättre vid högre hastigheter och mer extrema manövrar. Modellen baserat på framhjulen presterade konsekvent sämst.

Vid jämförelse med GNSS positionering visade det sig att de sensorbaserade modellerna gav bättre positionering de första 10 - 20 sekunderna. Därefter blev felet större på grund av fel i mätdatan samt modellerna. Att kombinera GNSS data och sensormätningarna med hjälp av ett Extended Kalman filter minskade denna drift vilket resulterade i en mer robust och tillförlitlig positionering gentemot att endast använda modellerna.

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Contributions

This thesis was carried out as a joint effort by Elias Pettersson and Anton Ahlsson. Both authors have contributed to the planning of the study, the data collection, analysis of results, and the writing and revision of the report.

Large parts of the project and the report have been produced together. Elias Pettersson has focused more on the rear axle and front axle models and the evaluation of the Kalman filter-based sensor fusion. Anton Ahlsson has worked more with the different bicycle models and the comparison between motion models and GNSS positioning. Although individual responsibilities were divided across these areas, all major decisions, interpretations and conclusions were developed together.

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Acronyms

ABS Antilock Breaking System.

ADAS Advanced Driver Assistance Systems.

DR Dead Reckoning.

EKF Extended Kalman Filter.

EPE End Point Error.

GNSS Global Navigation Satellite System.

GPS Global Positioning System.

IMU Inertial Measurement Unit.

KF Kalman Filter.

LiDAR Light Detection and Ranging.

MAE Mean Absolute Error.

RMSE Root Mean Square Error.

RTK Real Time Kinematic.

SAS Steering Angle Sensor.

WSS Wheel Speed Sensor.

1 Introduction

1.1 Background

Accurate and reliable vehicle positioning is a fundamental requirement for modern transportation systems and navigation applications. Positioning information is vital in a wide range of vehicle related functions such as route guidance, traffic monitoring, Advanced Driver Assistance Systems (ADAS) and autonomous vehicles [1]. In ADAS and autonomous driving applications, accurate and reliable positioning data enables functions such as lane keeping, collision avoidance and automated parking. These functions contribute to improved road safety as well as driving efficiency, and beyond individual vehicle level, improved positioning also improves traffic management systems. These systems can reduce congestion, improve transport efficiency and thereby lower environmental impact. Consequently, as research on ADAS and autonomous driving is rapidly advancing, the demand for precise and reliable positioning has never been more significant.

Since the introduction of Global Navigation Satellite System (GNSS) in the 1990s, satellite-based positioning has been used as a solution for determining vehicle position in many applications [2]. GNSS has served as a central component for vehicle positioning over the last three decades, however, its performance is highly dependent on the surrounding environment and signal conditions. GNSS interference is commonly observed in urban environments, particularly in dense high-rise areas, tunnels, bridges, industrial zones and harbors [3, 4]. Furthermore, recent years show an increasing risk of intentional GNSS interference linked to military or governmental activities [5].

These effects can lead to lowered positioning accuracy or complete loss of vehicle position. This is unacceptable in assisted or autonomous driving systems, since it could pose a safety risk to passengers, other road users and infrastructure. This is a major issue as the areas most affected by these challenges are characterized by critical infrastructure and high population densities. To address these limitations, vehicle-based sensors can be used to support positioning when GNSS measurements are degraded or completely unavailable.

To mitigate the limitations of GNSS and increase the position accuracy, vehicle-based sensors can be used to estimate vehicle position. For example, Wheel Speed Sensors (WSSs), Steering Angle Sensors (SASs) and Inertial Measurement Units (IMUs) can be used in Dead Reckoning (DR) methods to estimate vehicle position [6, 7]. The IMU contains motion sensors (accelerometers) and angular velocity sensors (gyroscopes) and by integrating their measurements the system can calculate the vehicle position over time [8]. Similarly, WSS and SAS measurements can be used to estimate vehicle

position through mathematical models which enable DR position estimates, independently of GNSS. The main challenge with these methods is the error sources from wheel slip, measurement errors, unequal wheel diameters, uneven road surfaces, skidding and changes in wheel diameters [1]. By fusing data from GNSS and the on-board sensors, it is possible to improve positioning accuracy and robustness, compared to relying on GNSS alone. This motivates investigation of DR methods with sensor data and sensor fusion approaches for vehicle positioning in GNSS challenged areas.

1.2 Research Questions and Purpose of the Study

The aim of this thesis is to investigate how wheel speed sensors and steering angle sensors can be utilized for dead reckoning position estimation, and how these estimates can be integrated with GNSS data using an Extended Kalman Filter (EKF) to improve heavy vehicle positioning accuracy and reliability. The following research questions are addressed:

- How does the selection and implementation of motion models influence the position error accuracy of wheel sensor-based dead reckoning position estimates?
- How does wheel sensor-based dead reckoning perform compared to standalone GNSS in terms of position error?
- To what extent can the position error be improved by fusing GNSS data and wheel sensor data, compared to only using wheels sensor data?

1.3 Limitations and Delimitations

This thesis is limited to vehicle data gathered on dry asphalt in favorable weather conditions. In these conditions, tire slip is expected to be minimal, allowing the motion models to be evaluated without any significant disturbances. Consequently, the results from the study cannot be assumed to be representative for other weather or road surface conditions such as rain, gravel or snow which would negatively affect the results.

The study is further limited to unloaded trucks. Vehicle mass as well as load distribution have major influence over the vehicle dynamics and tire behavior. Therefore, the results may not be transferable to trucks carrying payloads. This limitation is particularly relevant for dynamic models as their prediction is dependent on vehicle mass and forces. Additional testing would be needed to asses the performance of these models under varying loading conditions.

1.4 State of the Art

Approaches for DR vehicle positioning have been extensively studied, often with a focus on enhancing the odometry capabilities during GNSS outages. An early implementation of WSS data for DR is presented in [6], where four Antilock Breaking System (ABS) sensors were fused with Global Positioning System (GPS) in an EKF. Their approach continuously corrected the odometry-based estimation while GPS signals were available, and during longer GPS outages the proposed solution resulted in a maximum error of around 35 meters. Beyond using WSS for DR, many studies focus on the general usage of WSS, for example tire-road friction estimation [9], vehicle velocity estimation for ABS [10, 11], wheel slip and tire parameters [12] and investigating error sources when using WSS [13].

Recent studies have developed more complex methods, such as path tracking with maps and also incorporating more sensors such as Light Detection and Ranging (LiDAR) and cameras. For example in [14] WSS, yaw rate gyro and steering wheel measurements were used together with a kinematic model to minimize the position error. Their results showed an error within 1.5 meters after 100 meters traveled. In [8], GNSS, WSS and SASs was used with a kinematic bicycle model in an EKF and the results showed a estimation error of 13-50 meters after 30 seconds GNSS outage. Furthermore, many studies aim to mitigate the IMU drift during GNSS outages by incorporating WSS data, for example [7], where WSS and GNSS were used in a Kalman Filter (KF) to calculate IMU biases and errors. In [15], a differential drive model together with WSS and steering angle measurements was used to design a controller for autonomous driving based on DR. The controller followed a reference path and had a mean error for the stop position within 1 meter. Furthermore, in [16], sensors for steering angle, steering rate, yaw rate, and wheel speed were used together with a kinematic model to estimate position.

Today the focus has shifted towards artificial intelligence and machine learning and there are modern examples of how machine learning can be used to denoise IMU sensor data, see for example [17]. Others have used a deep learning-aided KF to fuse a low-cost angular accelerometer and a gyroscope [18]. This thesis aims to deepen the understanding of how different models perform when using DR for vehicle positioning. It further explores how wheel speed sensors work and their applicability in DR applications. Furthermore, this thesis contributes to the limited research of DR for heavy vehicles, where larger mass influences wheel slip and tire deformation during driving.

2 Theory

2.1 Motion Models

To estimate vehicle trajectories using DR, a motion model is required. This section presents the motion models investigated in this study. All models in this thesis define positive rotation as clockwise.

2.1.1 Unicycle Model

As an initial and simplified vehicle representation the unicycle model is introduced. The unicycle model is illustrated in Figure 1 and is commonly used in vehicle motion estimation and robotics literature [19]. In the model, θ represents the angular orientation of the unicycle and R refers to the radius of the turn. The angular velocity of the wheel is denoted by $\dot{\phi}$, with the corresponding linear velocity v given by $v = r\dot{\phi}$, where r is the radius of the wheel.

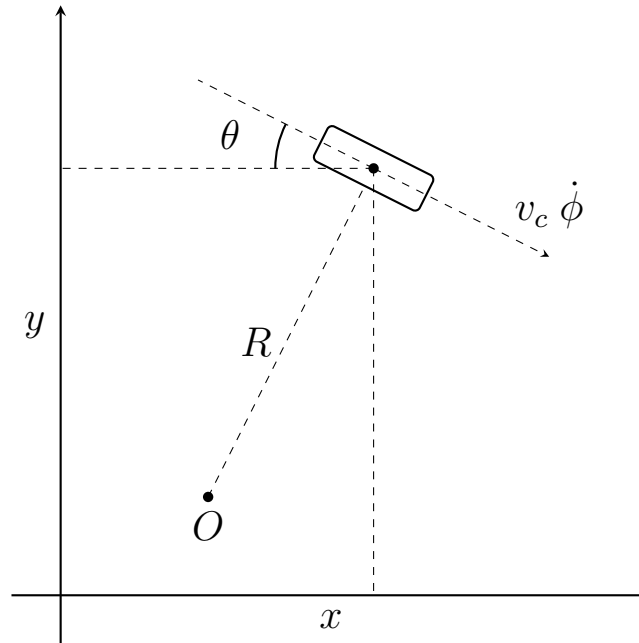


Figure 1. Unicycle model.

The changes in x and y -position are given by

$$\dot{x} = v \cos \theta \quad (2.1a)$$

$$\dot{y} = v \sin \theta \quad (2.1b)$$

where θ refers to the orientation of the vehicle. Note that the yaw rate

$$\dot{\theta} = \omega \quad (2.2)$$

is the change in vehicle orientation. This model provides a useful but simplified kinematic representation of vehicle motion and can be written in matrix form as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_c \\ \omega \end{bmatrix} \quad (2.3)$$

where v_c is the linear velocity and ω is the angular velocity, or yaw rate, of the vehicle. As (2.3) shows, this model requires the yaw rate as an input value. In this study, no sensor to directly measure this data was used. Note that the yaw rate can be calculated using the steering angle and the distance of the turning radius, but the turning radius is also unknown. Furthermore, yaw rate can be calculated using the steering angle, velocity and vehicle length. However, introducing these elements transitions the model from a unicycle representation to a bicycle model.

2.1.2 Kinematic Bicycle Model

A more comprehensive vehicle representation is the kinematic bicycle model, as illustrated in Figure 2. Unlike the unicycle model, the bicycle model could be implemented to estimate a vehicle trajectory using the sensor inputs available in this study. The kinematic bicycle is a well established motion model that is commonly used in vehicle trajectory estimation research, for example in [8, 15]. It is important to note that this model neglects slip effects, and therefore only describes the kinematic behavior of the vehicle.

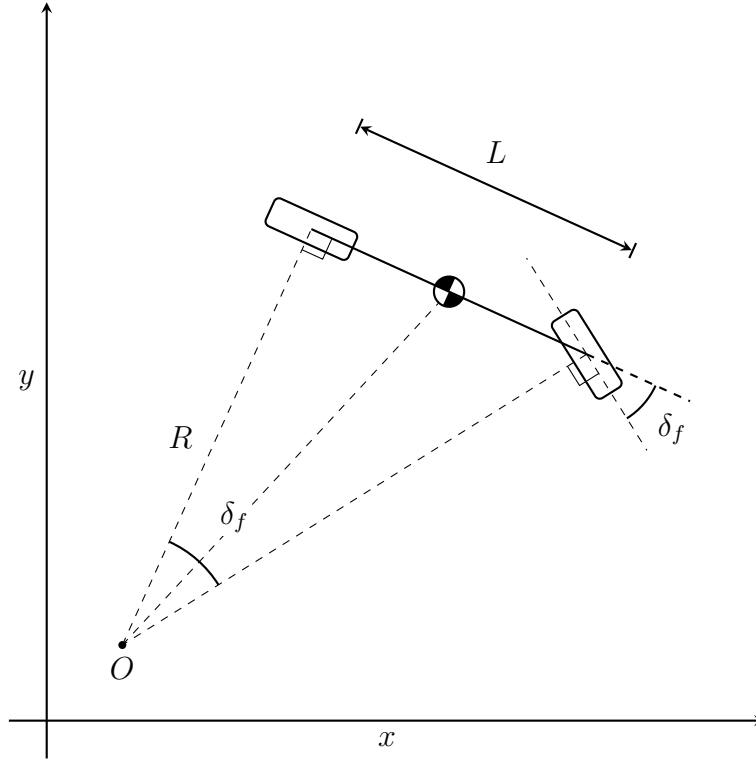


Figure 2. Kinematic bicycle model.

In this model, the steering angle of the front wheel δ_f is introduced and this changes the yaw rate that was previously defined as (2.2). The yaw rate is now expressed as

$$\dot{\theta} = \frac{v \tan(\delta_f)}{L} \quad (2.4)$$

where L is the wheelbase and δ_f is the steering angle. For small steering angles, the approximation $\tan(\delta_f) \approx \delta_f$ can be used. By combining (2.4) and the equations for motion from the unicycle model in (2.1) the system can be expressed by the following matrix

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta \\ \sin \theta \\ \frac{\tan \delta_f}{L} \end{bmatrix} v \quad (2.5)$$

which represents the nonlinear kinematic bicycle model in a compact state-space form.

This provides a more realistic description of vehicle motion compared to the unicycle model, while still remaining in the kinematic framework and therefore not accounting for wheel slip or forces.

2.1.3 Kinematic Bicycle Model with Slip

The presented kinematic bicycle model can be further extended by adding a slip angle to the vehicle's trajectory, as seen in Figure 3. The slip angle is calculated at the center of mass and allows the system to account for lateral tire slip, the type of slip that occurs when a vehicle is turning. The effect of lateral tire slip becomes increasingly significant at higher velocities, increased acceleration and during tighter cornering maneuvers. The inclusion of a slip angle is a standard extension of the previously presented model and is commonly used in vehicle dynamics literature [20, 21].

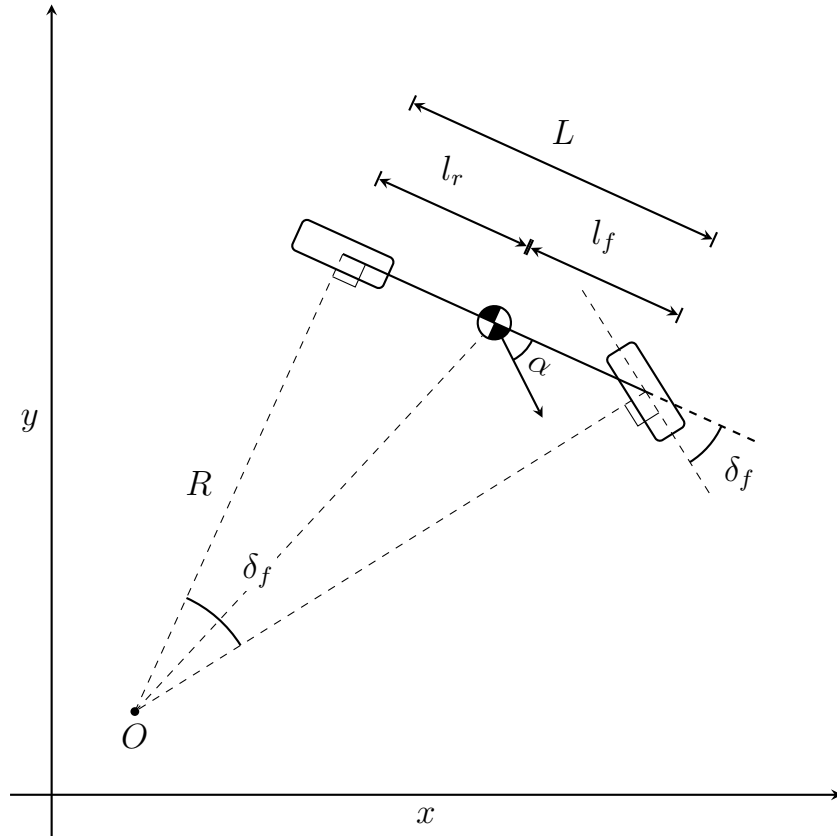


Figure 3. Kinematic bicycle model with slip.

For this model the slip angle α is given by

$$\alpha = \arctan \left(\frac{l_r}{l_f + l_r} \tan \delta_f \right) \quad (2.6)$$

where δ_f is the steering angle and l_f and l_r denote the distance from the vehicle's center of mass to the front and rear wheel axles, respectively. The slip angle and center of mass are then integrated into the original kinematic formula (2.5) creating

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta + \alpha) \\ \sin(\theta + \alpha) \\ \frac{v}{l_f + l_r} \cos \alpha \tan \delta \end{bmatrix} v \quad (2.7)$$

which represents the nonlinear kinematic bicycle model with slip in compact state-space form. This is a more realistic description of vehicle motion, accounting for lateral slip while remaining in the kinematic framework and ignoring acceleration and forces.

2.1.4 Dynamic Bicycle Model

As mentioned, the kinematic bicycle model does not account for acceleration and forces, which limits the model to low speeds and low acceleration scenarios. Therefore, to better capture realistic vehicle behavior, a dynamic bicycle model was investigated, see Figure 4. Dynamic models use the cause of motion such as forces, torques and inertia. This provides a more accurate and complex representation, but it also adds computational cost due to increased complexity and additional parameters, as shown in Figure 4. There are several different variants of dynamic bicycle models. These models differ in vehicle parameters, input data and assumption. The model used in this study is based on [21]. This model was selected because it relies on wheel-speed and steering-angle measurements, which are central to the scope of this study and commonly available in most vehicles.

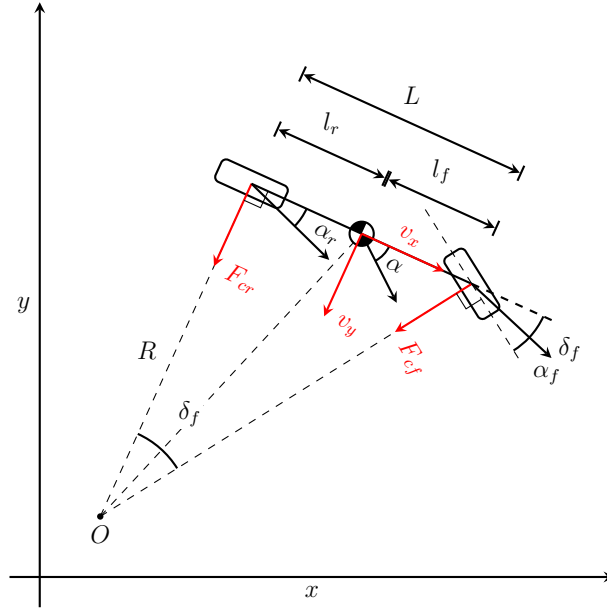


Figure 4. Dynamic bicycle model with slip.

In the dynamic model, the velocity is decomposed into longitudinal and lateral components. They are used to calculate change in position such as

$$\dot{x} = v_x \cos \theta - v_y \sin \theta \quad (2.8a)$$

$$\dot{y} = v_y \sin \theta + v_x \cos \theta \quad (2.8b)$$

where v_x and v_y denote the longitudinal and lateral velocities. This allows for acceleration and forces to be calculated. The change in velocity for the x -axis is given by

$$\dot{v}_x = \dot{\theta} v_y + a_x \quad (2.9)$$

where a_x is the acceleration in the x -direction. The velocity change in y -axis is calculated as

$$\dot{v}_y = -\dot{\theta} v_x + \frac{2}{m} (F_{cf} \cos \delta_f + F_{cr}) \quad (2.10)$$

where m is the mass of the vehicle and F_{cf} and F_{cr} are the lateral forces on the front respective rear tire. These forces are calculated using

$$F_{cf} = C_{\alpha f} \alpha_f \quad (2.11a)$$

$$F_{cr} = C_{\alpha r} \alpha_r \quad (2.11b)$$

where the front and rear tire slip angle α_f and α_r are multiplied by the cornering stiffness's $C_{\alpha f}$ and $C_{\alpha r}$ of the respective tire. The cornering stiffness is a constant but the slip of the front and rear tires vary during driving. The slip angles are calculated as

$$\alpha_f = \delta_f - \alpha - \frac{l_f \dot{\theta}}{v_x} \quad (2.12a)$$

$$\alpha_r = \alpha + \frac{l_r \dot{\theta}}{v_x} \quad (2.12b)$$

which is a combination of the steering angle, the slip angle of the vehicle and the center of mass along with the velocity in x -axis and change in yaw rate. Since the dynamic bicycle model represents the velocity using longitudinal and lateral components, the vehicle slip angle cannot be determined using (2.6) as in the kinematic bicycle model. Instead, the slip angle is given by

$$\alpha = \arctan\left(\frac{v_y}{v_x}\right) \quad (2.13)$$

and it describes the longitudinal slip of the vehicle body. The yaw rate is calculated by

$$\dot{\theta} = \frac{v_x}{l_f + l_r} \tan \delta_f \quad (2.14)$$

where the velocity in x -direction and the distances to the center of mass l_f and l_r is combined with the steering angle δ_f . The derivative of the change in yaw rate utilizes the distances to the center of mass and the lateral forces on the tires, such as

$$\ddot{\theta} = \frac{2}{I_z} (l_f F_{cf} - l_r F_{cr}) \quad (2.15)$$

where I_z is the yaw inertia

$$I_z = 0.5 m L^2 \quad (2.16)$$

which is a constant that depends on the mass and length of the vehicle.

2.1.5 Rear Axle Model

The previously presented bicycle models calculate vehicle motion and yaw rate using steering angle and wheel velocities. A vehicle can also be modeled using a rear axle model, commonly referred to as differential drive model. This vehicle model is solely based on the rotational velocities of the rear wheels, not requiring any steering measurement. This makes the model particularly attractive in scenarios where wheel speed data are more accurate than steering data or steering data are not available at all. The model originates from mobile robotics, where it is used to describe the motion of vehicles with independently driven wheels [19, 22]. It is also commonly applied in vehicle trajectory estimation [6, 15]. The model is presented in Figure 5 and represents vehicle motion using the velocities of the left and right rear wheels, denoted v_l and v_r respectively. Since there is no steering angle, the track width T becomes an important component in the calculation.

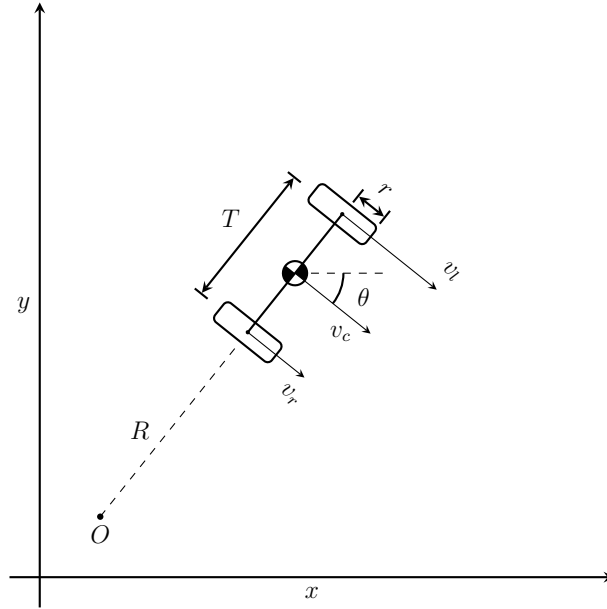


Figure 5. Rear axle model.

The motion of the center of mass of the vehicle is still described using the unicycle model introduced in (2.3). However, as previously mentioned there is no steering angle so the yaw rate is instead described by

$$\omega = \frac{v_r - v_l}{T} \quad (2.17)$$

where a larger velocity difference between the wheels results in a larger yaw rate. The velocity at the center of mass is obtained from the average of the the two wheel velocities

$$v_c = \frac{v_r + v_l}{2} \quad (2.18)$$

which represents the translational velocity of the vehicle. The motion in the x - and y -plane is described as

$$\dot{x} = v_c \cos \theta \quad (2.19a)$$

$$\dot{y} = v_c \sin \theta \quad (2.19b)$$

$$\dot{\theta} = \omega \quad (2.19c)$$

which allow vehicle trajectories to be estimated based on the rear wheel speed measurements. Similarly to the original bicycle model presented in Figure 2, this model describes vehicle motion kinematically, only using geometric relations. Consequently, factors such as tire slip, lateral forces and other dynamic effects are not represented in the model. This may negatively impact estimation accuracy during aggressive maneuvers or high speeds.

2.1.6 Front Axle Model

The front axle model is similar to the differential drive model, but here the wheels are steered, see Figure 6. The steering angles of the front wheels, δ_{fl} and δ_{fr} , differ from the nominal steering angle δ_f in the bicycle models due to the wheels turning at different angles during a turn. This allows the wheels to follow their own turning radius which reduces tire slip and dragging, and this geometric steering principle is known as Ackermann steering [23].

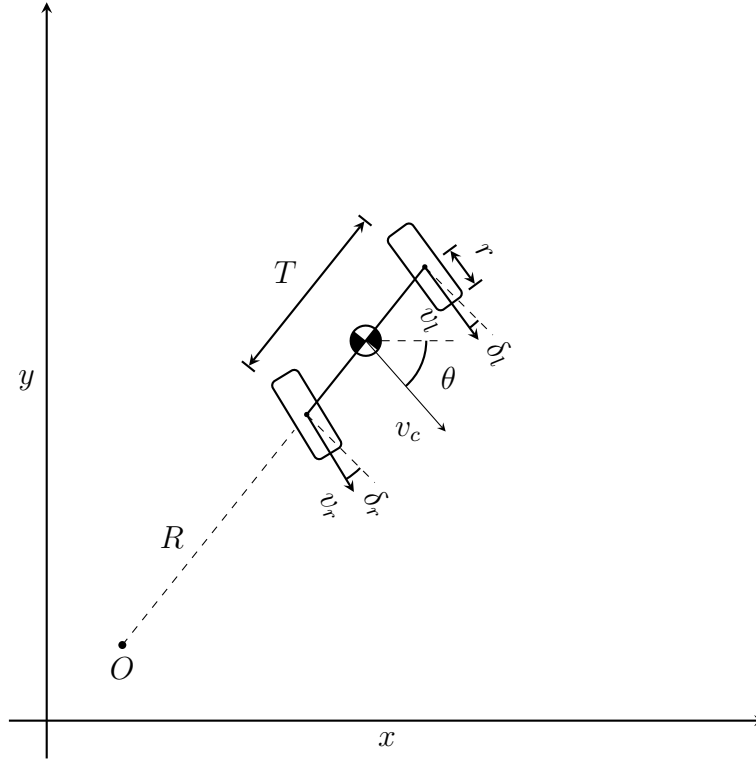


Figure 6. Front axle model.

For the front axle model, the wheel velocity components are described as

$$v_{l,x} = v_l \cos \delta_l \quad (2.20a)$$

$$v_{l,y} = v_l \sin \delta_l \quad (2.20b)$$

for the left wheel and

$$v_{r,x} = v_r \cos \delta_r \quad (2.21a)$$

$$v_{r,y} = v_r \sin \delta_r \quad (2.21b)$$

for the right wheel. The velocity components at the axle center are described by averaging the velocities. The velocities are given by

$$v_x = \frac{v_{l,x} + v_{r,x}}{2} = \frac{v_l \cos \delta_l + v_r \cos \delta_r}{2} \quad (2.22)$$

for the x -direction and

$$v_y = \frac{v_{l,y} + v_{r,y}}{2} = \frac{v_l \sin \delta_l + v_r \sin \delta_r}{2} \quad (2.23)$$

for the y -direction. The yaw rate

$$\omega = \dot{\theta} = \frac{v_{r,x} - v_{l,x}}{T} = \frac{v_r \cos \delta_r - v_l \cos \delta_l}{T} \quad (2.24)$$

is calculated as the difference in longitudinal wheel speeds. Similarly to the dynamic bicycle model the velocity is decomposed into longitudinal and lateral components, such as

$$\dot{x} = v_x \cos \theta - v_y \sin \theta \quad (2.25a)$$

$$\dot{y} = v_y \sin \theta + v_x \cos \theta \quad (2.25b)$$

and they are used to calculate the change in position. The change in orientation of the vehicle θ is

$$\dot{\theta} = \omega \quad (2.26)$$

which is consistent with the other models.

2.2 Sensors for Data Collection

2.2.1 Global Navigation Satellite System

GNSS is an umbrella term for global satellite positioning systems such as GPS, GLONASS and Galileo. These systems work by utilizing several satellites that all constantly broadcast their position at precise times [24]. A receiver then calculates how long it takes for each satellite's signal to arrive, and this allows the receiver to calculate the distance to each satellite and triangulate the receivers position [1].

The quality of GNSS data is highly dependent on the surrounding environment of the receiver. The signal can become degraded or lost if there are objects blocking the receiver from the satellite, this sort of signal blockage can result in an incorrect position or

no position at all [4]. GNSS signals can also suffer from multi-path errors, which occur when the signal bounces off of terrain or buildings which delays the signals travel time to the receiver [25]. These errors are illustrated in Figure 7 and they are most frequent in urban areas. Typically, GNSS provides accuracy within 5 meters [24].

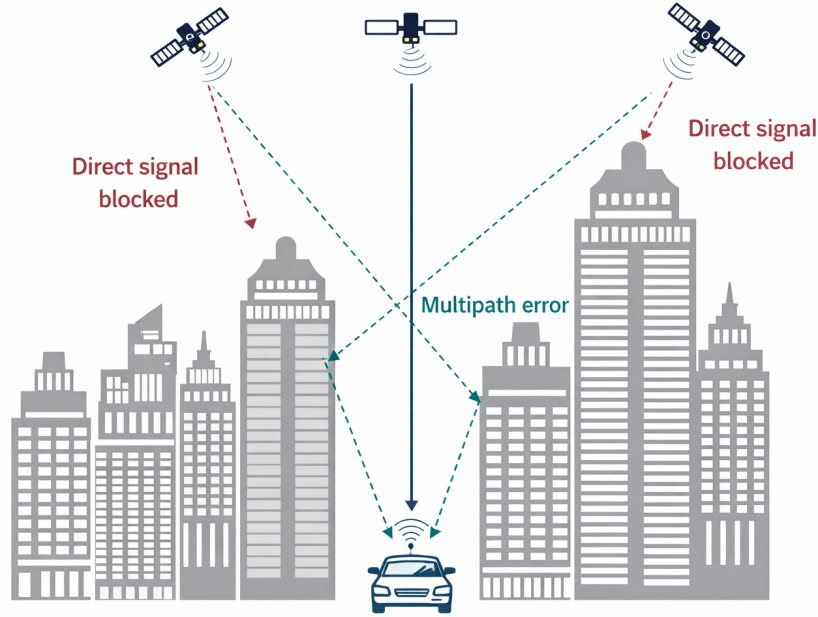


Figure 7. GNSS signal blockage and multipath effects in an urban canyon environment.¹

2.2.2 Real Time Kinematic Positioning

One way to increase the positioning accuracy and reliability is to use Real Time Kinematic (RTK) positioning, which is an improved GNSS technique that provides centimeter-level accuracy. RTK positioning uses observations from a local reference point with a known position together with measurements from a moving rover station. By comparing similarities of errors between the two receivers, it can correct position estimates and improve accuracy significantly compared to standalone GNSS [26]. RTK GNSS was used in this thesis to gather validation data to evaluate the performance of the positioning techniques explored in this thesis, as RTK measurements are accurate enough to be used as the ground truth trajectory.

¹This image was generated using Chat-GPT.

2.2.3 Wheel Speed Sensors

WSS measure the rotational speed of each wheel, which in turn can be used to estimate the velocity of a vehicle. They are typically integrated in most modern vehicles as they are a part of ABSs [6]. There are mainly two types of wheel sensors, passive and active, and in this project passive sensors were used. These sensors consists of two parts. The first part is a metal ring with evenly spaced holes, and the second part is the sensor which is a permanent magnet fixed in a position where its tip is close to the ring [27], see Figure 8.

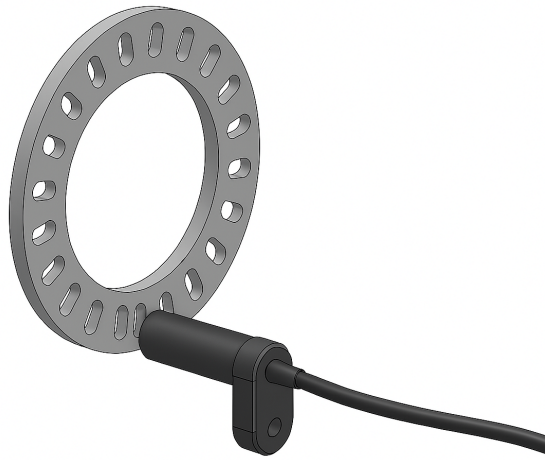


Figure 8. Passive wheel speed sensor.²

When a hole passes the sensor, it causes a change in the magnetic flux, which induces an alternating current voltage proportional to the wheel's speed. The resulting signal is processed and converted into a square wave to allow the system to count the pulses (or 'ticks') accurately. The ring used in this thesis had 100 holes, and each tick represents 3.6 degrees. The tick frequency, together with the radius of the wheel can be used to calculate how fast the wheel is traveling relative to the ground. WSSs measure rotational speed with high precision, factors such as wheel slip, uneven wheel diameters and road surface conditions degrade the accuracy of the derived vehicle speed over ground [1]. Active sensors work similarly, but instead of using rings with holes or teeth they use a magnetic encoder ring. The sensor detects the alternating magnetic poles, producing a signal with a frequency proportional to the vehicle speed [27].

²This image was generated using M365 Copilot. First a draft was hand-drawn by the authors, then it was refined using the AI tool to improve visual quality.

2.2.4 Steering Angle Sensors

The SAS is usually mounted in the steering wheel (hand wheel) and measures its angle. However, in this project, the SAS is mounted where the steering rack is connected to the wheel, see Figure 9. Each sensor measures the steering angle of its respective wheel relative to the coordinate system of the vehicle [28]. Wheel based sensors were selected over steering wheel based sensors as they collect data for each individual wheel's steering angle. The wheels have different steering angles due to the Ackermann geometry, which allows each wheel to follow its own turning radius and thereby reducing tire slip and dragging during turns. SAS have proven to be effective sensors, but they do have drawbacks [29]. SAS are affected by several error sources including deformations of steering components, sensor misalignment, calibration offsets and uneven road surfaces [29, 30].



Figure 9. Steering angle sensors are placed in the steering knuckles, marked by the red rings.³

³This image was generated using M365 Copilot.

2.3 Sensor Fusion

2.3.1 Kalman Filter

The theoretical description along with the equations presented in this section are based on [31]. For linear systems subjected to white Gaussian noise, the KF is an optimal recursive least squares state estimator. The KF is widely used in various different applications and commonly used in navigation systems [32]. The filter operates by combining data from multiple different sensors, a process called sensor fusion. By accounting for the given uncertainty associated with each measurement, the filter produces a statistically optimal estimate of the system state. Consider the linear system

$$\mathbf{x}_k = \mathbf{A}\mathbf{x}_{k-1} + \mathbf{B}\mathbf{u}_{k-1} + \mathbf{w}_{k-1} \quad (2.27a)$$

$$\mathbf{z}_k = \mathbf{C}\mathbf{x}_k + \mathbf{v}_k \quad (2.27b)$$

where $\mathbf{w}_k$ is the process noise and $\mathbf{v}_k$ is the observation noise, both assumed to be zero-mean Gaussian white noise with covariances $\mathbf{Q}_k$ and $\mathbf{R}_k$. The KF operates in two main steps, a prediction step and an update step. In the prediction step, the current state estimate $\hat{\mathbf{x}}_{k-1|k-1}$ is propagated forward in time using the system model, providing a prior estimate of the state. This estimate is given by

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{A}\hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}\mathbf{u}_{k-1} \quad (2.28)$$

where $\mathbf{A}$ is the state transition matrix that describes how the system evolves over time and $\mathbf{B}\mathbf{u}_{k-1}$ represents the the input signal's effect on the system. This creates a prediction of the new state estimate $\hat{\mathbf{x}}_{k|k-1}$ based on the previous state, before incorporating new measurements. Then the uncertainty of the predicted state is expressed by the covariance matrix, given by

$$\mathbf{P}_{k|k-1} = \mathbf{A}\mathbf{P}_{k-1|k-1}\mathbf{A}^T + \mathbf{Q}_k \quad (2.29)$$

where the term $\mathbf{A}\mathbf{P}_{k-1|k-1}\mathbf{A}^T$ represents the propagation of the previous uncertainty through the system dynamics. $\mathbf{Q}_k$ represents the process noise covariance and other uncertainties in the system, such as unmodeled dynamics. After the prediction step is complete the update step incorporates new measurements to correct the the predicted state estimate. By comparing the predicted estimate with the new measurement, the state prediction can be adjusted accordingly. This process starts by calculating the innovation

$$\tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{C}\hat{\mathbf{x}}_{k|k-1} \quad (2.30)$$

where $\mathbf{z}_k$ is the new measured state value and $\mathbf{C}$ is the measurement matrix. $\mathbf{C}\hat{\mathbf{x}}_{k|k-1}$ represents the predicted measurement, the result the sensors are expected to measure. The innovation represents the difference of the predicted measurement and the actual new measurement. Then the uncertainty of the innovation is calculated by the innovation covariance

$$\mathbf{S}_k = \mathbf{C}\mathbf{P}_{k|k-1}\mathbf{C}^T + \mathbf{R}_k \quad (2.31)$$

where $\mathbf{P}_{k|k-1}$ is the predicted state covariance and $\mathbf{C}\mathbf{P}_{k|k-1}\mathbf{C}^T$ represents how the state uncertainty is projected to the measurement space. $\mathbf{R}_k$ is the measurement noise covariance. Then, the Kalman gain is computed by

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{C}^T\mathbf{S}_k^{-1} \quad (2.32)$$

which determines how much the predicted state should be altered based on the new measurement. The calculation is based on the predicted covariance, the measurement matrix and the inverse innovation covariance. Using the Kalman gain an updated state estimate is calculated by

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\tilde{\mathbf{y}}_k \quad (2.33)$$

where this updated state estimate is given by the original predicted state estimate adjusted by the innovation weighted by the Kalman gain. Finally the updated covariance is computed so the process can continue according to

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k\mathbf{C})\mathbf{P}_{k|k-1} \quad (2.34)$$

where $\mathbf{P}_{k|k}$ is the updated state covariance.

2.3.2 Extended Kalman Filter

The standard KF is an optimal estimator for linear systems with Gaussian noise and to address the limitations regarding non-linearities and non-Gaussian noise, several variants have been developed. For example, the EKF, the unscented KF (UKF) and other,

see, for example, [31]. In the EKF, the system is linearized around the current mean and covariance. Consider the non-linear system

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + \mathbf{w}_{k-1} \quad (2.35a)$$

$$\mathbf{z}_k = h(\mathbf{x}_k) + \mathbf{v}_k \quad (2.35b)$$

where $\mathbf{w}_k$ is the process noise and $\mathbf{v}_k$ is the observation noise, both assumed to be zero-mean Gaussian white noise with covariances $\mathbf{Q}_k$ and $\mathbf{R}_k$. The prediction step for the EKF is first the predicted state estimate

$$\hat{\mathbf{x}}_{k|k-1} = f(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_{k-1}) \quad (2.36)$$

and the predicted covariance estimate is

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{Q}_{k-1} \quad (2.37)$$

where $\mathbf{F}_k$ is defined as

$$\mathbf{F}_k = \left. \frac{\partial f}{\partial \mathbf{x}} \right|_{\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_{k-1}} \quad (2.38)$$

which is the Jacobian of the nonlinear model. The filter is very similar to the KF, but as seen above the system is linearized before the update step, which is as follows

$$\tilde{\mathbf{y}}_k = \mathbf{z}_k - h(\hat{\mathbf{x}}_{k|k-1}) \quad (2.39a)$$

$$\mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k \quad (2.39b)$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{S}_k^{-1} \quad (2.39c)$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k \quad (2.39d)$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} \quad (2.39e)$$

where $\mathbf{H}_k$ is given by

$$\mathbf{H}_k = \left. \frac{\partial h}{\partial \mathbf{x}} \right|_{\hat{\mathbf{x}}_{k|k-1}} \quad (2.40)$$

which is the Jacobian of the observation model.

3 Method

To achieve the aim of the thesis, experimental data was collected by driving a truck at a test field in different driving scenarios. These scenarios included varying curvatures and speeds, lane change maneuvers, repeated driving on the same route at varying speeds and highway-like driving. The scenarios were selected to evaluate the performance and robustness of the methods during different operating conditions. The same truck was used for all tests.

The data was then preprocessed by interpolating the tick times to obtain exact measurements of wheel speeds. Selected parts of the data were then used to calibrate the model parameters, all models were calibrated on the same data. To evaluate their performance, the models were tested on separate validation data in different driving scenarios. An EKF was also implemented and evaluated using independent data.

3.1 Data Collection

This section describes the sensors and measurements used to collect the dataset used in the experiment. As mentioned, data was first collected from experimental runs on the test field providing data from different driving scenarios. The following sections describe how this data was used and preprocessed.

3.1.1 Wheel Speed Sensors

Figure 10 illustrates the sampled voltage from a test experiment. The dots represent the measured voltage from the WSS and the square wave represents the ticks, switching between 0 V and 5 V as the analog signal crosses zero.

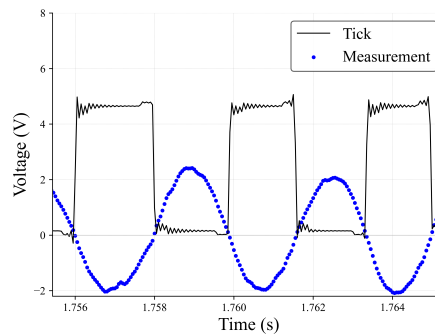


Figure 10. Example of raw sensor measurement and corresponding tick.

This discretization introduces a quantization error which can be observed in Figure 11. To address this error, linear interpolation was applied to estimate the time at which the measured voltage crosses zero.

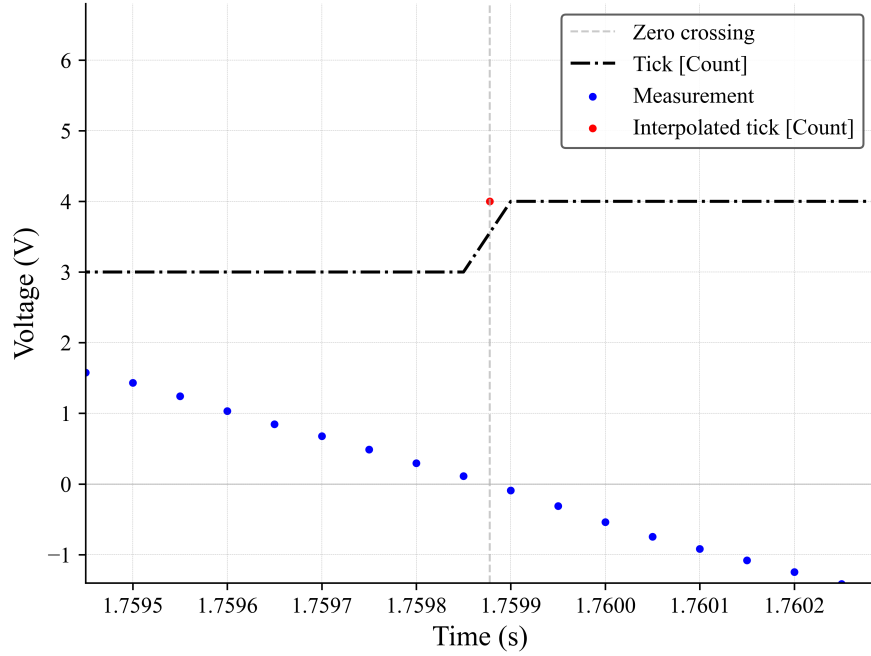


Figure 11. Measurement error when counting ticks.

Given two consecutive voltage measurements (t_k, V_k) and (t_{k+1}, V_{k+1}) where the signal changes sign, the zero-crossing time can be approximated as

$$t_{zero} = t_k - V_k \times \frac{(t_{k+1} - t_k)}{(V_{k+1} - V_k)} \quad (3.1)$$

where t_{zero} is represented by the interpolated tick in Figure 11. This approximation assumes linear variation of the signal between samples, which is a simplification. A higher sampling frequency would reduce this approximation error. This approximation allows the timing of every tick to be very precise which greatly improves the accuracy of the velocity data, as demonstrated in Figure 12. On the left side, the velocity is calculated based on the interpolated data and on the right side the velocity is based on raw tick data. This greatly increases the performance of the models as an accurate velocity has a major impact.

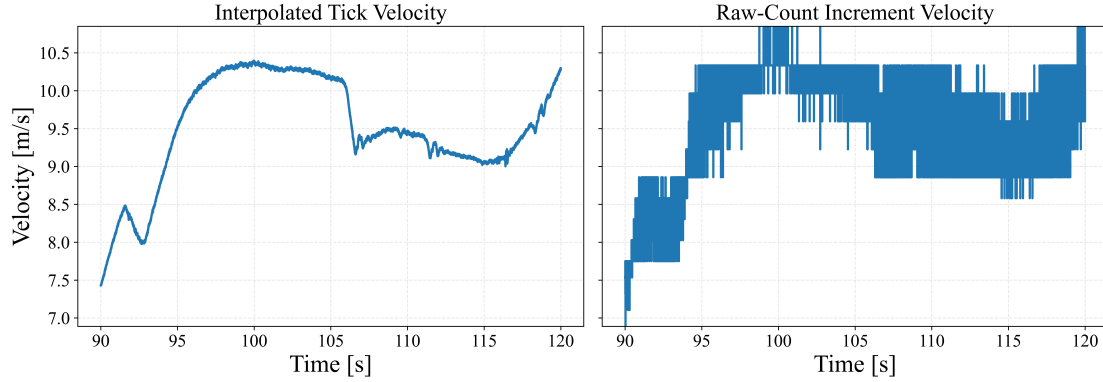


Figure 12. Velocity estimation based of interpolated data to the left and velocity estimation based on raw data to the right.

The WSS was used by all motion models to collect vehicle velocity data. The models used the data in different ways depending on the structure and inputs of the model. For the the bicycle models the wheel speeds from all the wheels were averaged to estimate a single central velocity. This make the data consistent with the bicycle models, as they only use one velocity. In the rear axle model each of the rear wheels were used separately, and for the front axle model, the front wheels' velocities were used separately.

3.1.2 Steering Angle Sensor

For the bicycle models, the steering angles of both front wheels were combined into an average angle for the vehicle. This approach was chosen as the testing truck (along with most other vehicles) has Ackermann steering, meaning that the inner wheel in a turn has a greater turning angle than the outer wheel. Only using one wheel would create a systematic error, as the model would oversteer when turning one way and understeer turning the other [20]. Using both angles separately was not an option as the model only needs one angle-input. For the front axle model the steering angles were treated separately and each angle was used in the model. The rear axle model is based only on wheel velocities, and does not need the steering angles.

3.1.3 GNSS Measurements and RTK Reference

GNSS data was accessed from the vehicle's internal system. This data was used as it represents the information available during standard operations. To collect the data used for validation of the motion models and GNSS, an RTK receiver was mounted on the

testing truck. The RTK was connected to base stations supplied by Lantmäteriet, which are connected to the reference system SWEREF 99, the official reference system for positioning in Sweden [33].

3.2 Calibration

The track width T and the wheel radius r are difficult to determine or measure accurately in a real test scenario. The effective track width depends on the vehicles lateral center of mass and the exact location of pressure on the wheels as well as the distance between them. Not knowing this introduces a large uncertainty in the parameter value. To address this uncertainty, a series of initial calibration tests were conducted. The available data was divided into calibration and evaluation subsets. The calibration data was used to estimate the model parameters, and the evaluation data was used to test the models.

3.2.1 Distance Per Tick

First, the truck was driven in a straight line while maintaining a speed as constant as possible. Second, the RTK trajectory was converted from longitude-latitude coordinates to local Cartesian x and y coordinates. Then an initial DR path was calculated using the rear axle model and a first approximate guess of track width and distance per tick for each wheel. These first parameter settings were received from vehicle documentation. To improve the estimation and adjust the trajectory to the RTK distance, the driven distance was divided with the accumulated ticks for each wheel,

$$d_{left} = \frac{\Delta s_{RTK}}{N_{left}} \quad (3.2a)$$

$$d_{right} = \frac{\Delta s_{RTK}}{N_{right}} \quad (3.2b)$$

where d_{left} and d_{right} is the distance per tick for each wheel, Δs_{rtk} is the traveled distance and N_{left} and N_{right} is the tick count for that run. Furthermore, the distance per tick was optimized to fit as well as possible by minimizing the distance to the RTK path. Here a variable parameter was added to both d_{left} and d_{right} , which was optimized to minimize the position error. The resulting trajectory's can be viewed in Figure 13.

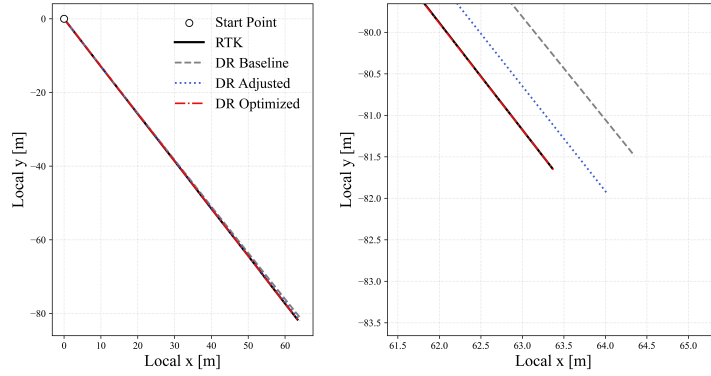


Figure 13. Plot of path from RTK , DR Baseline, DR Adjusted and DR Optimized. To the right is a zoomed in plot on the trajectory end point.

3.2.2 Track Width

After calibrating the distance per tick, these values were kept fixed and the track width was calibrated separately. The distance per tick calibration requires straight driving, but the track width calibration requires turning motion. Figure 14 shows the results of the calibration. The left plot compares RTK reference trajectory with DR using the baseline track width and the optimized track width. The right plot illustrates the optimization process, where different track widths and initial heading were evaluated to determine the combination which gave the lowest Root Mean Square Error (RMSE). The wheelbase L was provided in the vehicle documentation.

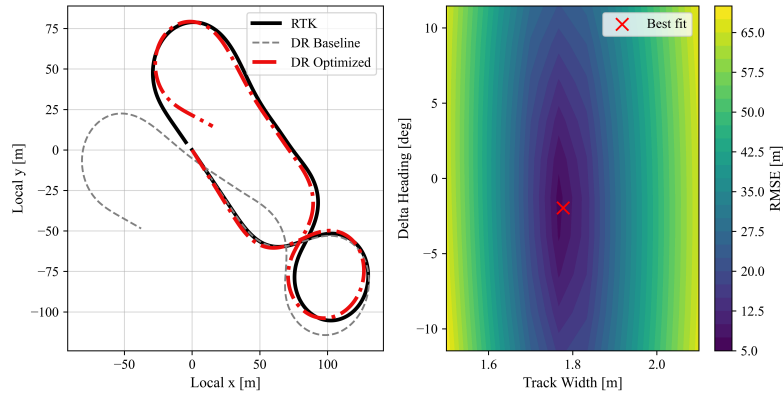


Figure 14. The data used for calibration to the left, and the RMSE for different track widths and initial headings to the right.

3.2.3 Wheel Stiffness

In the dynamic bicycle model, multiple constants were used, and all of these constants had known values except for the cornering stiffness of the front and rear wheels $C_{\alpha f}$ and $C_{\alpha r}$. Since these parameters were not available from the vehicle manufacturer they had to be estimated.

This estimation was completed using a grid search approach. An initial interval between 100 000 and 600 000 N/rad was set for both the front and rear cornering stiffness values. Within this interval 25 candidates were evenly spaced out for both parameters, resulting in a total of 625 possibilities. The combinations were evaluated by implementing them in the dynamic bicycle model and comparing the estimated trajectory to the RTK trajectory. The performance for the parameter combinations were evaluated by minimizing the RMSE. The results from this estimation can be seen in Figure 15.

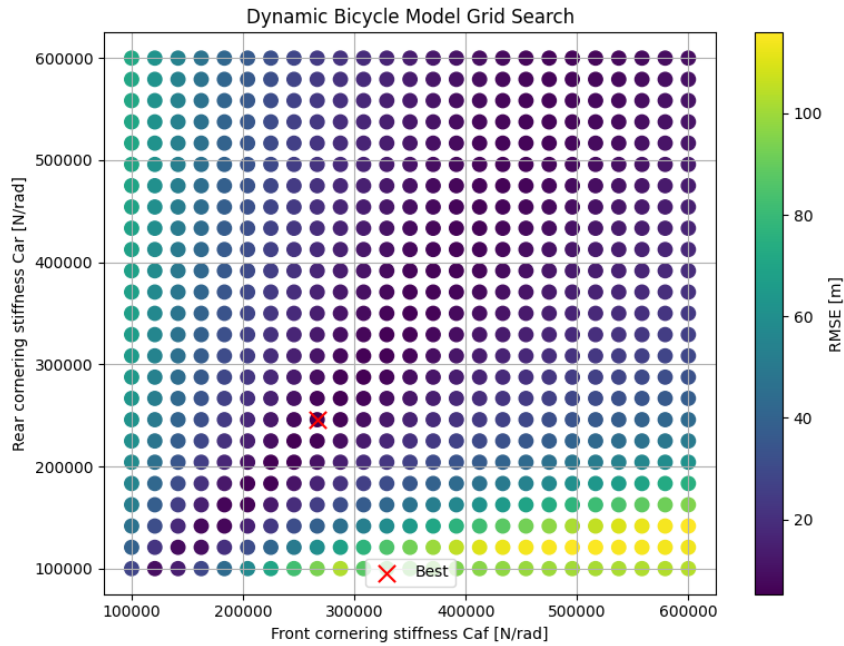


Figure 15. Cornering stiffness calibration.

After the initial optimization step, a second grid search was performed using a smaller interval centered around the combination with the smallest RMSE from the previous step. This allowed the parameter values to be estimated in greater detail, resulting in a more accurate estimate. The best values from this estimation were then implemented in the dynamic bicycle model for the study.

3.3 Sensor Fusion

The states x , y and θ were estimated in the EKF, but the selection of other system parameters was less obvious due to observability limitations. For example, wheel slip causes the WSS to overestimate the distance traveled compared to GNSS. Such errors may come from multiple sources, such as incorrect wheel radii or track width. The filter adapts the parameters during driving, but it cannot distinguish what exactly causes the error and therefore does not know which model parameters to adjust. An initial approach was to use the rear axle model in the EKF and use the following state vector

$$\mathbf{x} = [x \ y \ \theta \ d_l \ d_r \ T]^T \quad (3.3)$$

where

$$v_l = \phi_l^{tick} d_l \quad (3.4a)$$

$$v_r = \phi_r^{tick} d_r \quad (3.4b)$$

$$\dot{\theta} = \frac{v_r - v_l}{T} \quad (3.4c)$$

and ϕ^{tick} represents ticks per second, and d is the distance per tick. This allows the filter to calibrate the distance per tick for each wheel and the track width during driving. The states d_l , d_r and T were modeled as slowly varying parameters by assigning them low process noise, which restricted the EKF to small real time corrections. The track width T should be tuned when turning, and the distance per tick d_l and d_r should be tuned during straight driving, due to observability. Therefore, the filter was designed with two process noise matrices which were switched between depending on yaw rate. For straight driving the process noise was set to

$$\mathbf{Q} = \text{diag}(0.01, 0.01, 0.001, 5 \times 10^{-10}, 5 \times 10^{-10}, 10^{-14}) \quad (3.5)$$

allowing d_r and d_l to move more compared to T . For cornering maneuvers it was set to

$$\mathbf{Q} = \text{diag}(0.01, 0.01, 0.001, 5 \times 10^{-10}, 5 \times 10^{-10}, 5 \times 10^{-8}) \quad (3.6)$$

to allow T to move more freely. To determine when to use which process noise to use, the yaw rate was used with a threshold for straight driving within ± 0.02 rad/sec.

3.4 Performance Measures

In order to evaluate the positioning performance of the motion models, several error metrics were used. The predicted vehicle trajectory from each motion model was compared to the RTK positioning trajectory, which was used as a reference for validation. Mean Absolute Error (MAE) was used to measure the average position error over an entire trajectory. For each time step i , the Euclidean distance between the motion model estimate and the RTK position was calculated. The MAE calculation is defined by

$$MAE = \frac{1}{N} \sum_{i=1}^N \sqrt{(x_i^{model} - x_i^{RTK})^2 + (y_i^{model} - y_i^{RTK})^2} \quad (3.7)$$

where N is the number of compared samples. This means all errors contribute linearly to the metric. This creates an intuitive metric that represents the average trajectory errors. RMSE was also used and similar to MAE, RMSE evaluates the error by comparing points along the entire trajectory. The difference is that RMSE places greater emphasis on larger errors, by squaring the error before averaging it. RMSE is given by

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N [(x_i^{model} - x_i^{RTK})^2 + (y_i^{model} - y_i^{RTK})^2]} \quad (3.8)$$

where N is the number of compared samples. Both MAE and RMSE are commonly used to evaluate model performance, but the metrics emphasize different aspects of the trajectories [34]. In MAE all errors are equally weighted and contribute linearly, while in RMSE, larger errors will dominate the metric. This characteristic makes RMSE particularly useful for identifying models that accumulate a substantial drift over time. This can be very useful but it can also be problematic as a small number of large deviations can skew the metric. Since the aim of the motion models is to minimize drift over time, RMSE was used for the calibration of track width and cornering stiffness because it penalizes larger position errors. In addition to MAE and RMSE, the End Point Error (EPE) was also used as a performance measure. EPE is calculated as

$$e_{end} = \sqrt{(x_{end}^{model} - x_{end}^{RTK})^2 + (y_{end}^{model} - y_{end}^{RTK})^2} \quad (3.9)$$

where x_{end} and y_{end} corresponds to the last position in the trajectory. The EPE gives a clear picture of how much the total drift has accumulated by the end of the run. When comparing GNSS with DR the absolute position error was used to illustrate the position error at a given time.

4 Results and Discussion

4.1 Model Performance

4.1.1 Dead Reckoning Comparison

Selected parts of the validation data were used to represent different maneuvers, velocities and accelerations, allowing the models to be evaluated under varying dynamics. Five different motion models were tested in the experiments. The models were given an initial heading obtained from the first RTK coordinates. This was done as the motion models require an initial vehicle orientation.

Experiment 1 consisted of a relatively narrow turn followed by a wider turning maneuver. The vehicle velocity was between 9 - 13 m/s with a small but relatively constant acceleration. The distance was around 340 meters, and the results are presented in Figure 16.

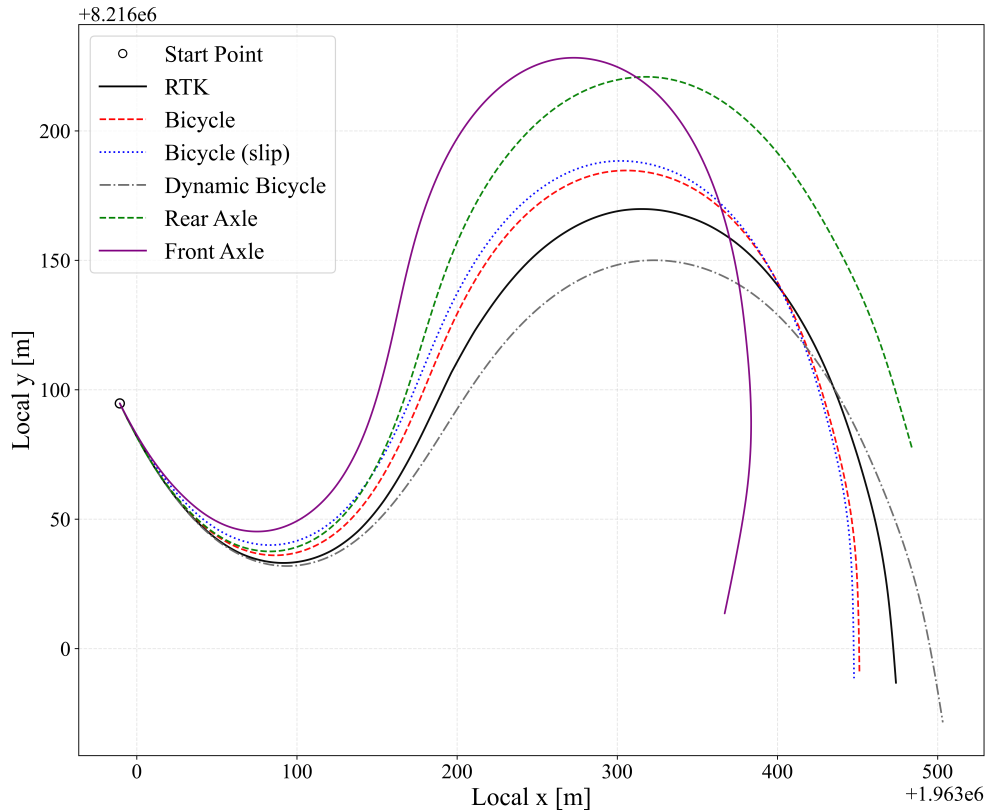


Figure 16. Experiment 1: Trajectory estimation for all models.

The results from experiment 1, seen in Table 1, show that the kinematic bicycle model without slip performed best, achieving the lowest score in all the performance measurements. The kinematic bicycle model with slip performed just slightly worse than the kinematic model. The dynamic bicycle model also produced somewhat accurate results, but drifted more than the kinematic models over time. In contrast, both the rear axle and front axle models struggled significantly early in the estimation and also accumulated a substantial drift over time.

Table 1: Performance measurement results for experiment 1.

Metric	Kinematic [m]	Slip [m]	Dynamic [m]	Rear Axle [m]	Front Axle [m]
RMSE	6.39	8.11	9.42	25.80	28.43
MAE	5.47	7.25	7.47	19.80	24.28
EPE	11.86	13.49	16.94	46.83	56.65

The results show that simpler models, such as the kinematic bicycle model, can outperform more advanced models. This could be explained by the characteristics of the experiment, with low velocity and acceleration, the slip effects become relatively small. Consequently, the additional complexity introduced by slip and dynamic components may not necessarily improve the estimation in such conditions. Furthermore, advanced models with more parameters risk overfitting to the calibration data, which makes them more sensitive to parameter uncertainties and noise. This could reduce their performance on validation data, especially when vehicle dynamics have a small impact. It is worth noting that the kinematic bicycle model with slip did not outperform the standard bicycle model. This indicates that the slip effects in experiment 1 were too small to justify the added complexity in more advanced models.

As mentioned, both axle models show large error in all metrics, and there could be multiple reasons for this. The models do not consider longitudinal wheel slip, and this could be a major reason for inaccuracies in the axle models. Since the yaw rate is calculated by the difference in wheel speed for the wheels, the longitudinal wheel slip affects the turning motion. If there are greater slip effects on one wheel, it over- or underestimates the steering angle, and this is observed for both models which struggle to capture the turning motion in the experiment. These effects are not relevant for the bicycle models, as they rely on a separate steering angle input.

Experiment 2 consisted of a segment with a constant heading and was performed at a velocity of 15 m/s - 20 m/s with relative even velocities in the end and beginning, and a burst of acceleration after 15 seconds. The distance traveled was around 522 meters, and the results are illustrated in Figure 17.

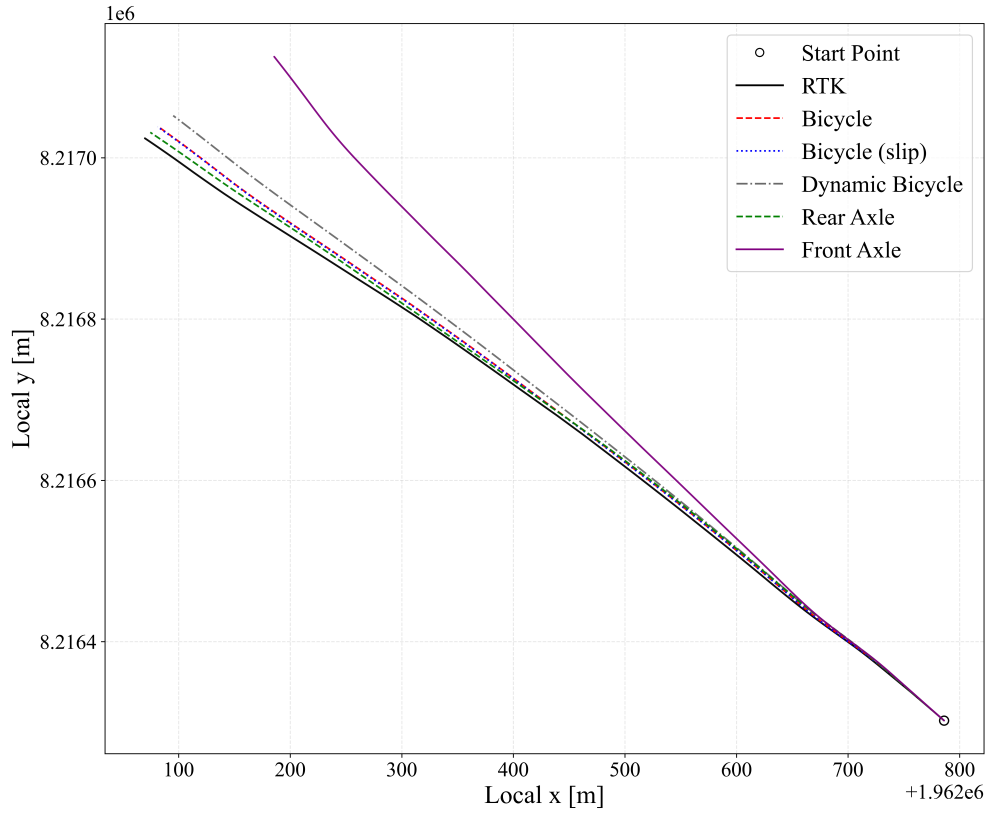


Figure 17. Experiment 2: Trajectory estimation for all models.

The performance metrics from experiment 2 are presented in Table 2. They show that all models, with the exception of the front axle model, were able to estimate the vehicle trajectory with high accuracy. The rear axle model performed the best with the lowest RMSE, MAE and EPE. The kinematic models also provided accurate trajectory estimations that showed small errors and closely followed the RTK reference. The dynamic bicycle model once again performed worse than the kinematic models, however, the errors were still relatively low.

Table 2: Results for experiment 2.

Metric	Kinematic [m]	Slip [m]	Dynamic [m]	Rear Axle [m]	Front Axle [m]
RMSE	4.09	3.91	8.69	2.74	33.19
MAE	3.22	3.04	6.71	2.45	24.12
EPE	9.65	9.38	19.43	4.52	78.97

These results are similar to experiment 1, but in experiment 2 there was a higher velocity and almost no steering changes. During such maneuvers, the trajectory will largely be determined by longitudinal factors, which reduces the importance of lateral modeling dynamics, which favors simpler models. The dynamic model is more sensitive to noise, and therefore performs worse. On the other hand, the rear axle model that solely relies on wheel velocity data performs very well. As discussed earlier the yaw rate is derived from difference in wheel velocities, and since in this experiment it can be assumed that the wheel slip was roughly the same for both wheels, meaning the yaw rate could be accurately calculated. The front axle model again produced substantially larger errors compared to the other models. This indicates that the assumptions in the model are not sufficient to accurately describe the vehicle motion.

Experiment 3 consisted of a significant turning maneuver combined with major changes in vehicle velocity. The truck started the experiment at about 20 m/s, decelerated rapidly to 8 m/s while navigating the turn and then accelerated back to 20 m/s upon exiting the turn. The total distance traveled was around 390 m. The results are illustrated in Figure 18.

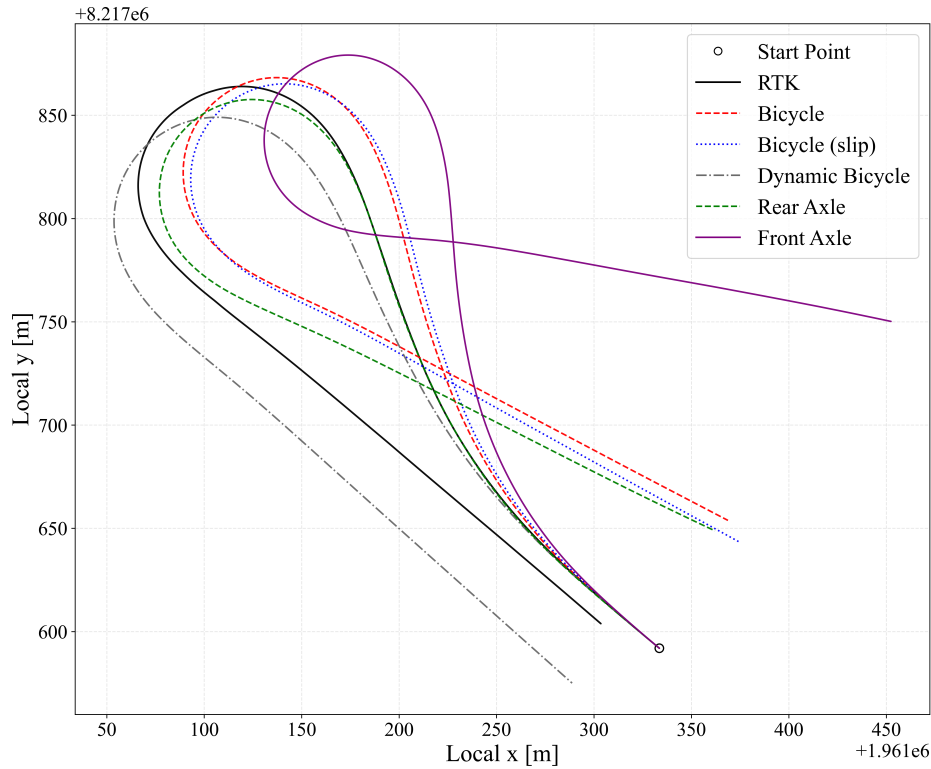


Figure 18. Experiment 3: Trajectory estimation for all models.

Experiment 3 challenged the models with a large turn at relatively high velocity combined with deceleration and acceleration. Here, the advantages of the dynamic model become evident, with superior results in both the RMSE and EPE metrics, see Table 3.

Table 3: Results for experiment 3.

Metric	Kinematic [m]	Slip [m]	Dynamic [m]	Rear Axle [m]	Front Axle [m]
RMSE	16.80	18.26	11.17	13.49	45.63
MAE	13.04	14.59	9.91	9.22	36.65
EPE	42.19	41.62	16.60	37.90	107.14

Experiment 3 was the most demanding of the three experiments, combining substantial velocity changes with a large turning maneuver. During these conditions there were clear differences compared to previous experiments. The kinematic bicycle models that previously performed well showed difficulty following the rapid acceleration and turning, accumulating larger errors. The front axle model once again performed the worst results in all metrics.

The rear axle drive produced the second lowest RMSE and the lowest MAE. This is an interesting observation as it suggest that the rear axle model had a relatively small average error throughout the majority of the experiment, but accumulated larger errors towards the end. Since RMSE penalizes larger deviations more compared to MAE and the EPE was twice as large compared to the dynamic model, the results indicate that the rear axle model experienced greater drift during the end of the experiment. In contrast, the dynamic model experienced less of a drift, but for the majority of the trajectory had a larger absolute error compared to the RTK reference. This observation highlights the value of considering both RMSE and MAE when evaluating vehicle trajectory estimation performance. RMSE is more sensitive to larger deviations, like accumulated drift, MAE provides a measure and absolute error throughout the trajectory. This allows drift error characteristics of the models to be identified. The improved performance for the dynamic bicycle model is most likely due to the model’s ability to account for lateral vehicle dynamics and tire forces. The same reasoning can be used for the decreased performance for the bicycle models, during sharp turns and acceleration the assumptions in the kinematic models are not valid.

The results from the three experiments demonstrate that the benefits of including more advanced vehicle dynamics become important as driving conditions become more demanding. The simpler kinematic models performed as good, if not better, than dynamic model in simpler maneuvers.

4.1.2 Model Performance at Different Velocities

To test how the models perform during different velocities, the truck was driven in the shape of an eight. The lap was driven at different speeds, and the resulting trajectories were evaluated for the different models. For the kinematic bicycle model the results are presented in Figure 19. Here it is clear that the model performs worse when the velocity increases.

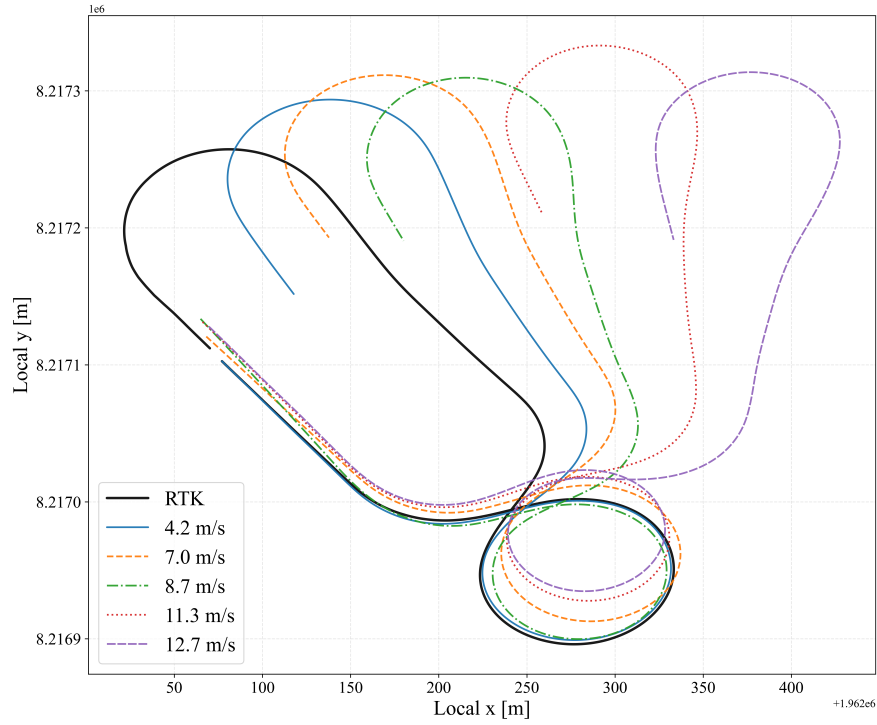


Figure 19. Kinematic bicycle model results for different velocities.

The metrics for the kinematic bicycle model are presented in Table 4. The results show a clear relationship between vehicle velocity and performance of the kinematic bicycle model. As the velocity increased from 4.2 m/s to 12.7 m/s all performance metrics increased considerably every step. An interesting thing to note is that at the beginning of each run, the model has very accurate positioning estimates. This indicates that the DR method has accurate performance for a limited duration, but as seen in the results, it starts to accumulate a large drift after the first circle. The higher velocity, the more it drifts, which greatly affects the latter parts of the run.

Table 4: Kinematic bicycle model results for different velocities.

V_{mean} [m/s]	RMSE [m]	MAE [m]	EPE [m]
4.6	21.07	14.97	31.78
7.0	27.11	19.52	48.22
8.7	38.93	27.66	65.53
11.3	63.72	45.14	105.91
12.7	86.56	61.47	137.38

These results are consistent with the results from the previous experiments. Showing that the assumptions underlying the model become less valid and functional as the vehicle dynamics become more demanding. At higher velocities, effects such as tire slip and lateral dynamics cannot be modeled by a kinematic model. The results show that even at relatively low velocities the error is substantial, even though the velocity is lower than in experiment 1 and 2. The reason for the low accuracy in these measurements is most likely the demanding turning maneuvers. This indicates that even at low velocities a kinematic model can struggle estimating vehicle motion if the maneuvering is demanding. The results for the bicycle model with slip are presented in Figure 20. Similar to the standard bicycle model, the estimated trajectory's drift further from the RTK reference as the velocity increases. This indicates that the model accumulates a substantial drift during challenging maneuvers, that increase with increased velocity.

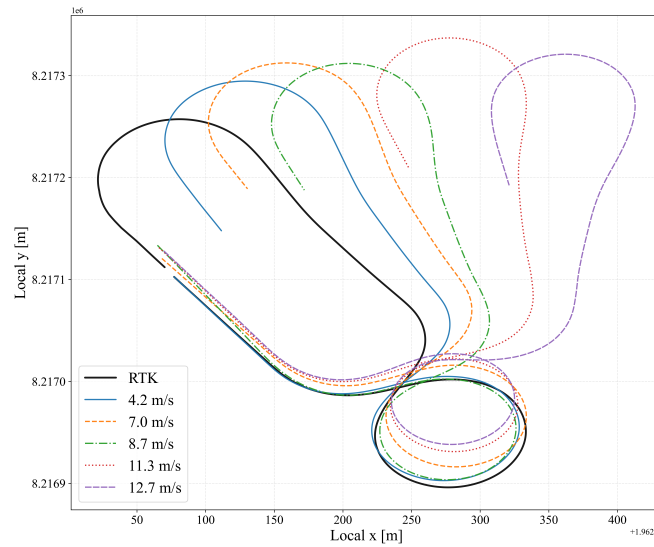


Figure 20. Kinematic bicycle model with slip results for different velocities.

The results presented in Table 5 support this observation as all error metrics increase with velocity. Overall the kinematic bicycle model with slip performs marginally better than the standard bicycle model, improving performance by a few meters on average. This shows that the addition of a slip angle slightly improves the positioning accuracy, but it is not enough to capture the vehicle motion accurately.

Table 5: Kinematic bicycle model with slip results for different velocities.

V_{mean} [m/s]	RMSE [m]	MAE [m]	EPE [m]
4.2	18.49	14.00	28.07
7.0	24.76	18.89	44.10
8.7	35.87	26.10	61.14
11.3	60.31	43.87	100.69
12.7	82.58	59.79	131.55

The trajectories for the dynamic bicycle model are shown in Figure 21. As seen, the model follows the reference relatively accurately for all velocities.

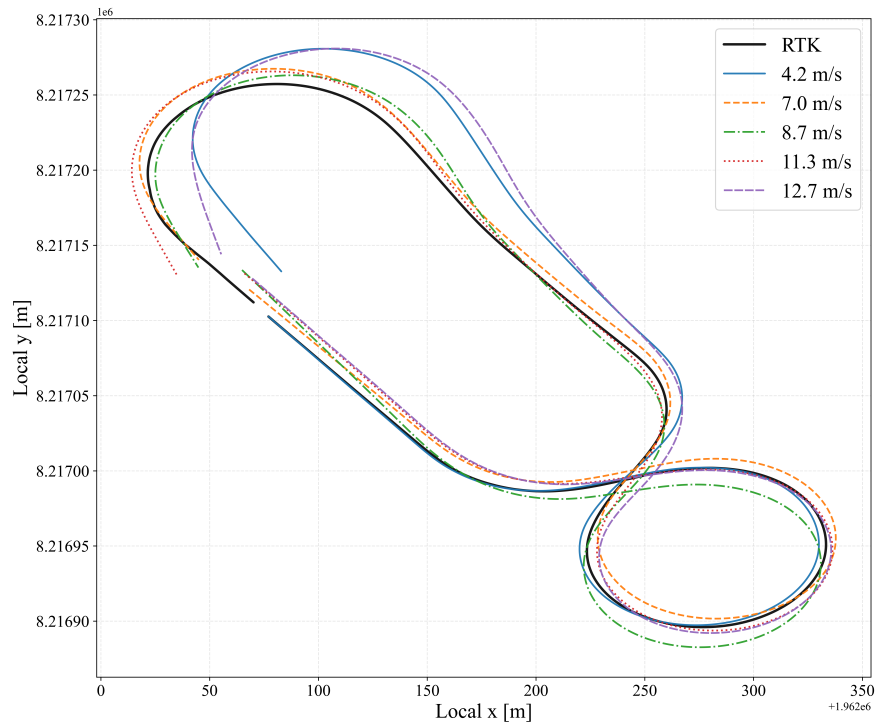


Figure 21. Dynamic bicycle model results for different velocities.

The results from the dynamic bicycle model differs greatly from the kinematic models. As shown in Table 6 the dynamic bicycle model maintains a relatively consistent level of accuracy across all the velocities. Unlike the kinematic models there is no clear drift that increases with velocity and all the runs resulted in an RMSE and MAE under 10 meters. The model still deviates slightly from the RTK reference this result still illustrates that the model is able to accurately capture the vehicle motion during harder driving conditions. This is in line with the way the model is built, incorporating lateral vehicle dynamics and tire forces.

Table 6: Dynamic bicycle model results for different velocities.

V_{mean} [m/s]	RMSE [m]	MAE [m]	EPE [m]
4.2	9.62	7.57	12.42
7.0	6.56	5.80	10.75
8.7	6.89	5.94	10.80
11.3	5.84	4.70	17.83
12.7	8.99	6.83	14.95

These findings are consistent with the results from earlier experiments. The dynamic model performs well when the turning maneuvers are stronger and is not as impacted by higher velocities or velocity changes. This supports the previous claim that the inclusion of lateral dynamics and tire forces in the modeling process becomes increasingly important when estimating vehicle trajectories during more demanding driving conditions.

The results for the rear axle model are illustrated in Figure 22. This model shows accurate results for almost all velocities, which was unexpected since it performed poorly in the first experiments. It can be observed that the results only start to drift more aggressively at velocities above 11 m/s. Since this is a kinematic model that does not consider slip, it was expected to perform similar to the bicycle model with increasing drift at higher velocities, but this experiment show that the rear axle model is more robust. This does not align well with the previous results, suggesting that the rear axle model performed worse compared to the bicycle models. The characteristics of the experiment must however be considered, where this data is collected at quite narrow turns, whereas the earlier experiments were at much wider turns. The longitudinal slip effects on the calculation of yaw rate was discussed earlier, in this experiment the model successfully captures the yaw rate in different velocities. It is interesting to note that during this experiment, the velocities were kept as constant as possible, which would reduce the longitudinal slip effect, and thus provide an accurate calculation of yaw rate.

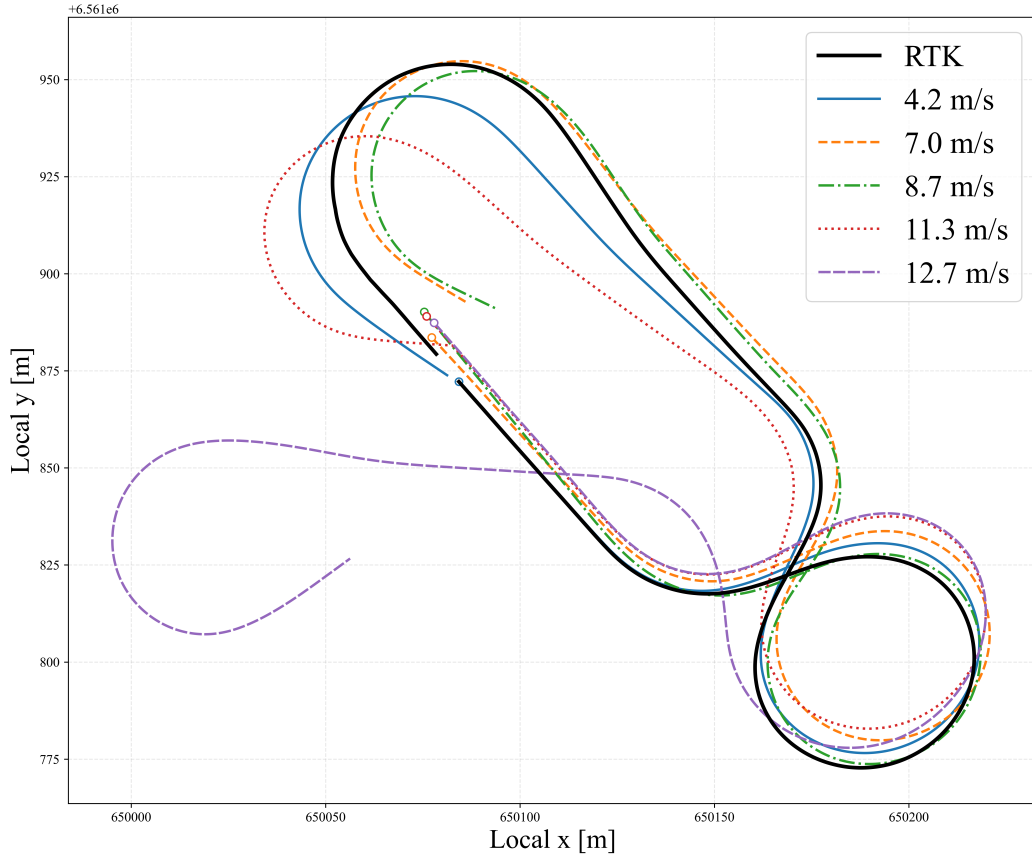


Figure 22. Rear axle model results for different velocities.

It can be observed that for the lowest velocity $V_{mean} = 4.2$ m/s, the model actually performs worse compared to driving at $V_{mean} = 7.0$ m/s, see Table 7. The explanation for this could be that at low speed, the WSS registers fewer ticks which would increase the quantization error.

Table 7: Rear axle model results for different velocities.

V_{mean} [m/s]	RMSE [m]	MAE [m]	EPE [m]
4.2	8.2	6.3	20.3
7.0	6.1	5.6	9.4
8.7	5.0	3.6	12.0
11.3	20.5	15.0	20.3
12.7	68.9	47.0	78.6

For the front axle model, the previous results show that this model does not perform very well, and this can also be observed for this test, illustrated in Figure 23.

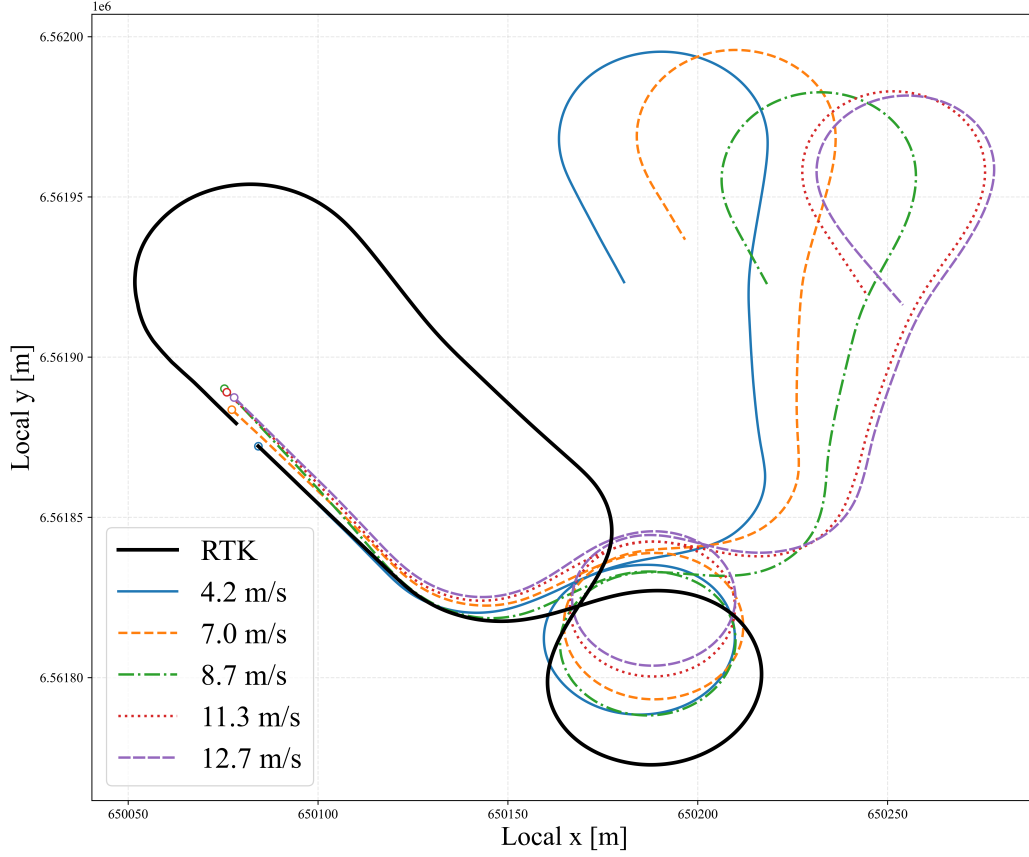


Figure 23. Front axle model results for different velocities.

The front axle model does perform well for the first part of the experiment, but as seen, it drifts off when driving out of the smaller circle, and the higher the velocity, the more it drifts. The metrics are presented in Table 8. Since the vehicle is driven by the rear wheels, there is potentially more longitudinal slip effects on them, and this could motivate using a front axle model. If there is no force driving the wheels, the slip effects should not be as large. Unfortunately, the experiments and model used in this thesis cannot answer whether this is the case. One reason for the poor performance of the front axle model is that the front wheels experience a lot of lateral slip effects. This could affect the yaw rate calculation and this causes the model to drift. As seen in the results, the trajectories are in large parts similar to the RTK trajectories, but at some points it over or underestimates a turn, causing the rest of the trajectory to be wrong.

Table 8: Front axle model results for different velocities.

V_{mean} [m/s]	RMSE [m]	MAE [m]	EPE [m]
4.2	69.0	50.8	111.24
7.0	72.8	52.2	129.9
8.7	82.8	58.4	146.1
11.3	96.1	69.2	169.6
12.7	99.1	72.5	176.3

4.2 GNSS vs. Dead Reckoning

The GNSS data from experiment 1 contained some errors, which did not affect the model evaluation, but made it unsuitable for comparing DR with GNSS trajectories. Therefore, for this comparison, a different time interval from the same data set was selected, which is illustrated in Figure 24. To compare the DR approach with GNSS positioning, the dynamic bicycle model was used throughout all experiments. This was due to it showing accurate positioning in most situations in earlier presented results. The DR trajectory is initialized at the first RTK position and it captures the first turn accurately, but after the second turn it start to drift. The GNSS measurements are generally close to the RTK trajectory, and the positioning error seems more constant.

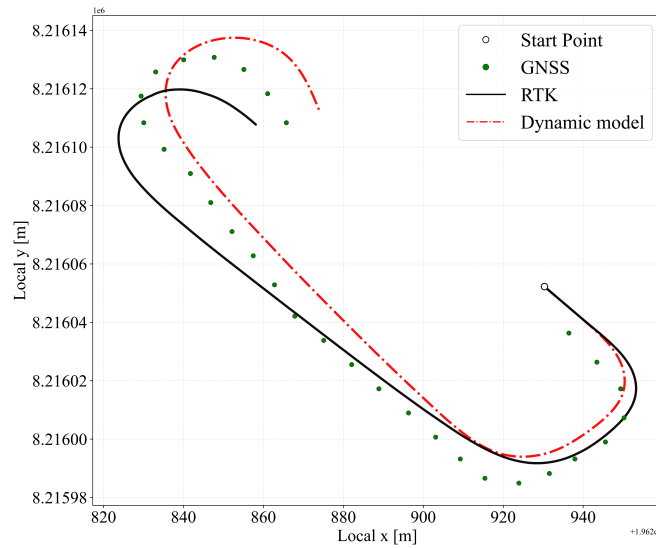


Figure 24. Alternative experiment 1: Results for dynamic bicycle model and GNSS.

To investigate when the DR positioning starts to drift and how it performs compared to GNSS positioning, the cumulative RMSE and the absolute positioning error over time is illustrated in Figure 25. The cumulated RMSE for the DR approach is below the cumulative RMSE for GNSS for about 20 seconds. This indicates that the DR method is more accurate compared to GNSS for 20 seconds, but Figure 25 also shows the absolute error, which illustrates that already after 10 seconds the positioning error is worse compared to GNSS. However, the absolute error for the GNSS varies around 6-8 meters for the entire run, and the absolute error for the DR approach is below 8 meters for around 17 seconds.

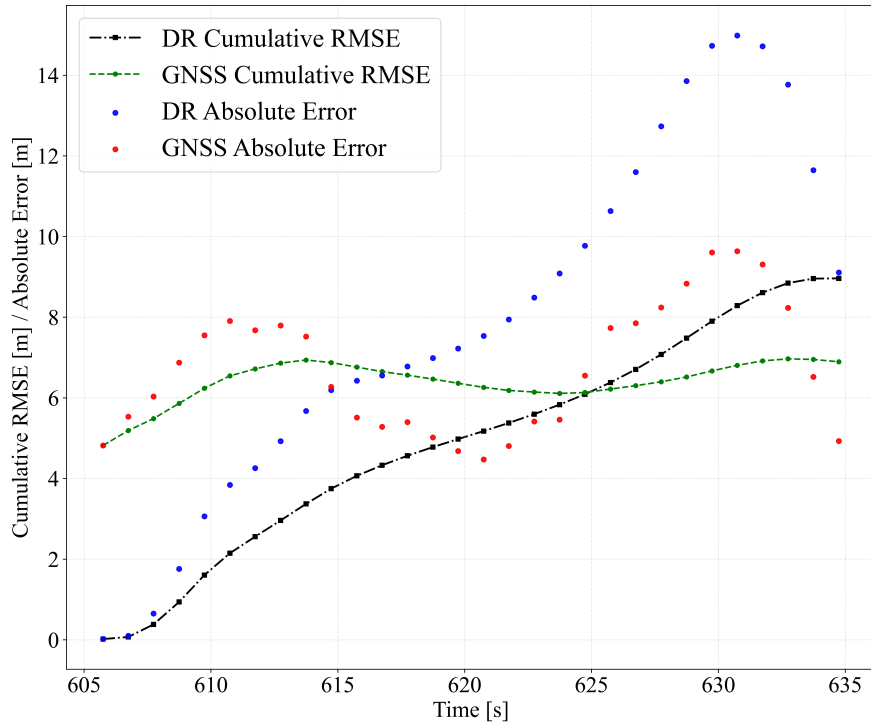


Figure 25. Alternative experiment 1: Cumulated RMSE and Absolute Position Error.

The results from experiment 2 are illustrated in Figure 26. This is the same data as used in experiment 2 in earlier results. The experiment consisted of straight driving for about 522 meters. The results illustrate that the dynamic model DR trajectory slowly drifts over the entire run, and that the GNSS is more accurate.

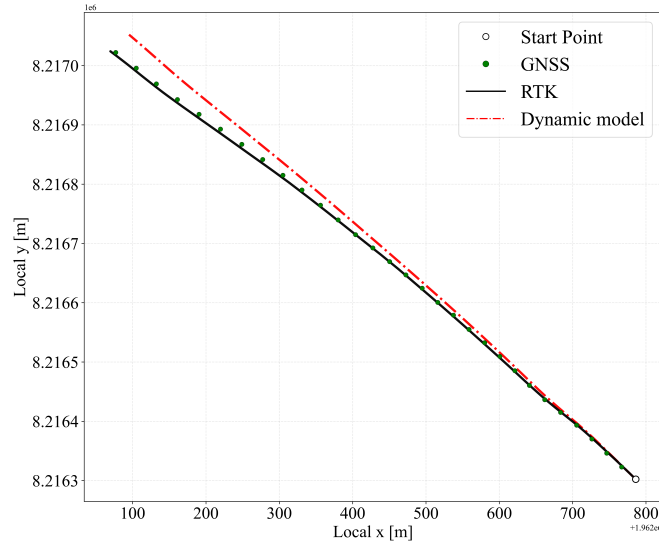


Figure 26. Experiment 2: Results for dynamic bicycle model and GNSS.

This is also indicated by the cumulative GNSS and absolute positioning error plot, illustrated in Figure 27. The cumulative RMSE for the dynamic model is again more accurate for around 20 seconds. The absolute positioning error does however show that after around 15 seconds the absolute error for the dynamic model is greater compared to the GNSS error. The GNSS error keep a rather constant absolute error of around 3 meters, whereas the dynamic model drift to almost 20 meters after 30 seconds.

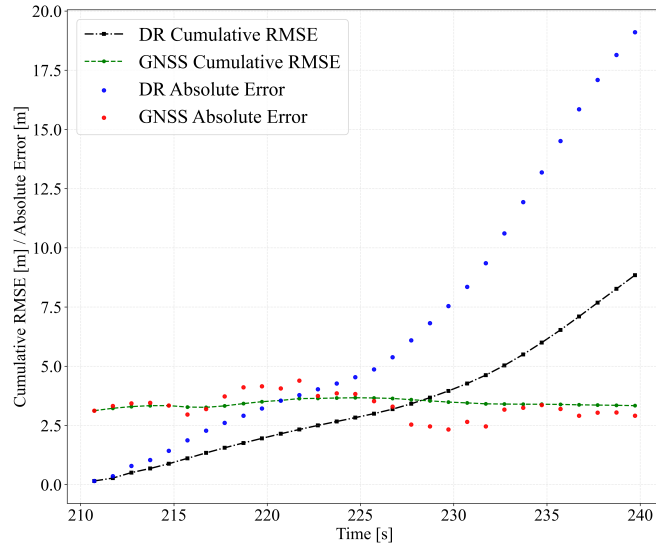


Figure 27. Experiment 2: Cumulated RMSE and Absolute Position Error.

Lastly, the results from experiment 3 are presented in Figure 28. Again, the trajectory plot indicates that the GNSS positioning is generally more accurate, and the DR positioning does not really capture the turning motion.

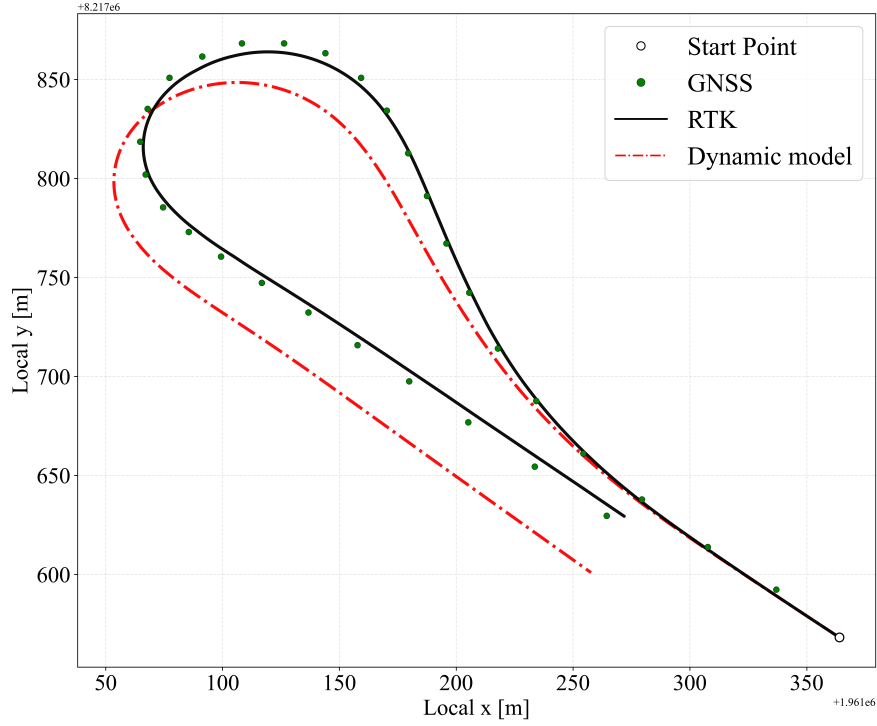


Figure 28. Experiment 3: Results for dynamic bicycle model and GNSS.

This is also evident when looking at the cumulative RMSE and the absolute error results, in Figure 29. The cumulative RMSE for the GNSS positioning is steady at around 4 meters, and the DR error is below it for about 10 seconds. The absolute error for GNSS is again quite stable at 4-5 meters, but it drifts to 16 meters for the DR approach.

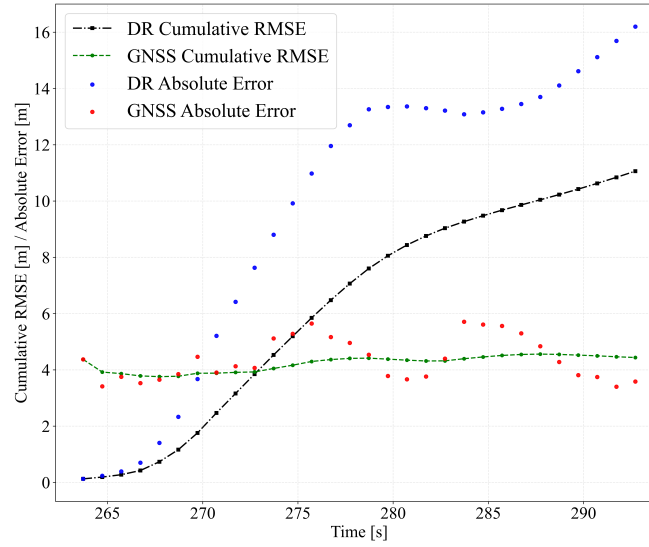


Figure 29. Experiment 3: Cumulated RMSE and Absolute Position Error.

To summarize these findings, the presented DR approach gives more accurate position estimates compared to standalone GNSS for about 10-20 seconds. As mentioned throughout the thesis there are many error sources affecting the calculations when using these sensors, and this explains the DR drift. In these experiments the dynamic bicycle model was used since it performed accurately in most driving scenarios, but as seen it still has difficulties to keep to the RTK ground truth after only a few seconds. However, it should be noted that the experiments were conducted in a rural environment where GNSS conditions are favorable. The gap in performance between GNSS and DR may be different in more challenging GNSS conditions.

4.3 EKF Performance

4.3.1 Performance for Turns

To evaluate the EKF an additional experiment was conducted, where the filter was first tuned during both straight driving and turning, and then evaluated compared to just DR with the model. The results for the first experiment can be viewed in Figure 30. The distance driven during the GNSS outage was around 310 m, and there was an acceleration from 8 m/s to 14 m/s. The EKF RMSE was 5.5 meters and EPE was 10.3 meters during the GNSS outage. This can be compared with the DR baseline which had an RMSE of 11.6 meters and an EPE of 20.2 meters.

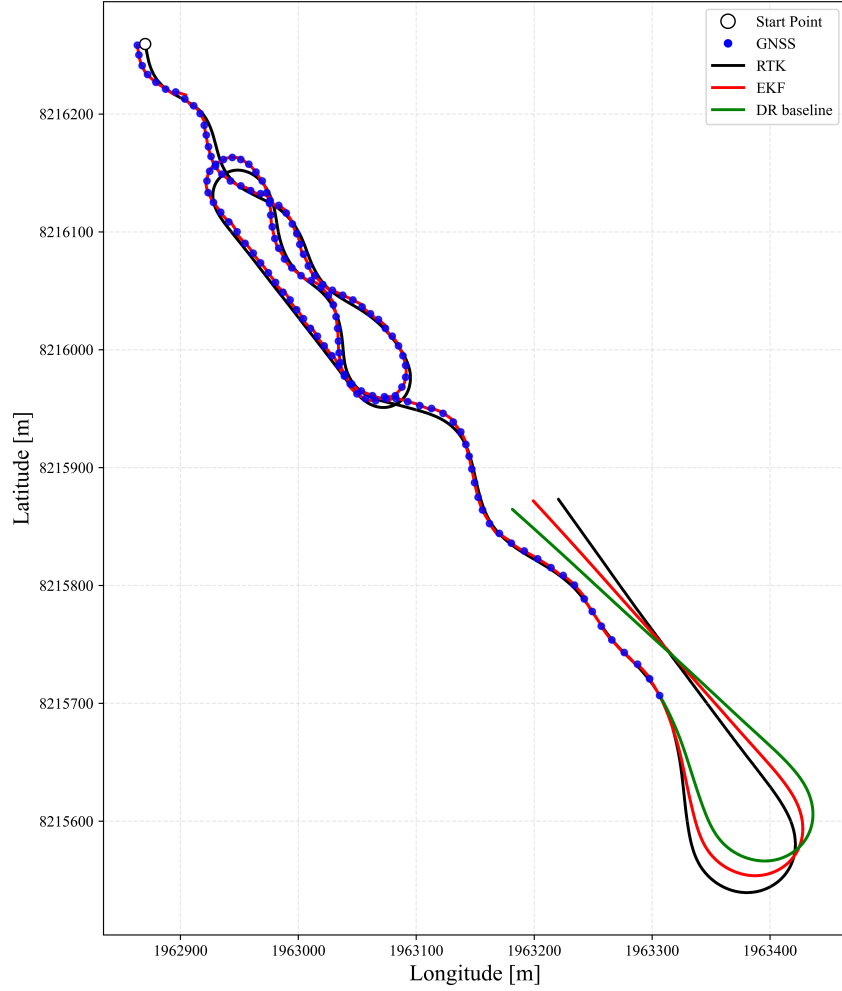


Figure 30. Comparing calibrated EKF (red line) with DR model path (green line).

For the second experiment presented in Figure 31, the distance driven during the GNSS outage was around 320 m, and there was a deceleration from 10 m/s to 8 m/s and then an acceleration to 14 m/s. The EKF performed better compared to the baseline DR path, but the improvements were not as large as for the first experiment. In Figure 31 it is clear the the EKF follows the RTK path closer, but then it deviates about as much as the baseline at the end of the turn. The EKF RMSE was 7.0 meters and EPE was 20.0 meters during the GNSS outage. This can be compared with the DR baseline which had an RMSE of 9.3 meters and an EPE of 25.7 meters. The presented filter thus makes the position predictions better, but when there is a GNSS outage the results still deviate after a short period. As mentioned before, this is due to various error sources which are not considered in the filter.

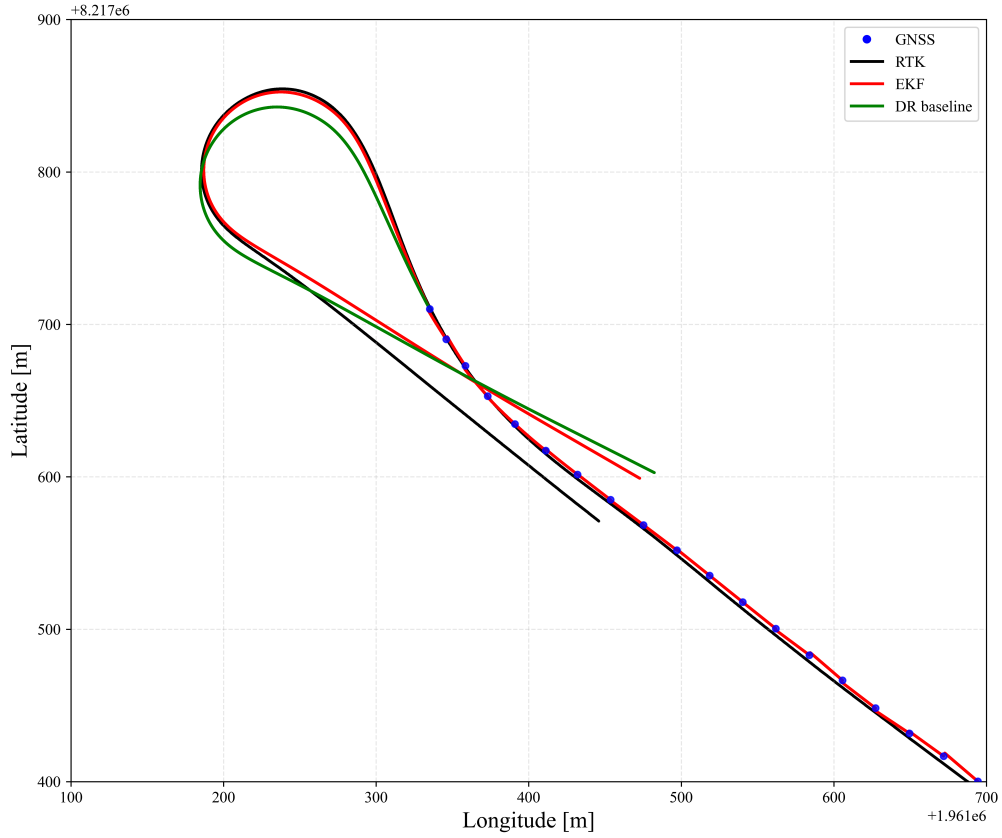


Figure 31. Comparing calibrated EKF (red line) with DR model path (green line).

4.3.2 Performance for Lateral Movement

To investigate how well the EKF captures lateral motion, such as lane changes, it was calibrated using 300 meters of driving data with GNSS measurements. Then a GNSS outage was simulated for around 200 meters, see Figure 32. The RMSE during GNSS outage was 3.18 meters and the EPE was 5.82 meters, and as seen in Figure 32 the lateral movement is quite well captured by the EKF. There is still an error of a few meters, but it seems as the model captures the small turns during a lane change.

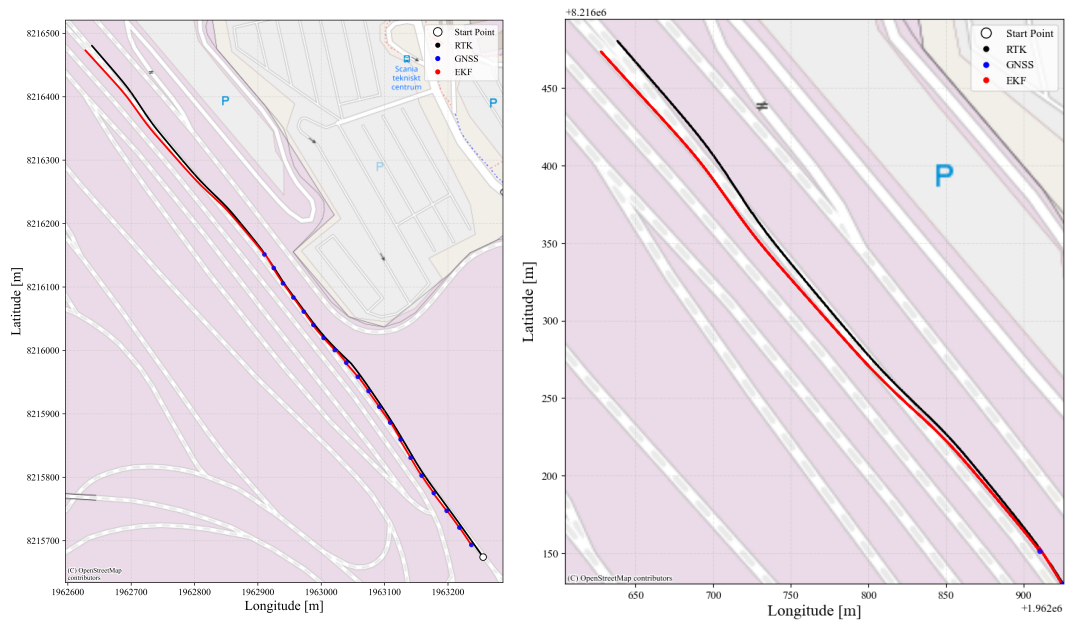


Figure 32. EKF performance during lane change, zoomed in to the right.

5 Conclusions

The first research question asked how the selection and implementation of motion models would influence the accuracy of sensor-based DR positioning. The results show that there is not a definite answer to the question, since the models investigated all have advantages and disadvantages, making them useful in different scenarios. The results for the kinematic bicycle model showed that it performed accurately in many situations, but deteriorated rapidly at higher velocities. The bicycle model with slip generally gave more accurate results compared to the kinematic bicycle model, but also had problems handling higher velocities. The dynamic bicycle model positioning was more accurate at higher velocities and more aggressive turning, but it requires complex computations and it is dependent on the wheel stiffness calibration. The axle models showed varying results, but generally the rear axle model performed better. The rear axle model seems to perform accurately when there is small longitudinal slip and when driving with constant velocity. The front axle model performed worse in all metrics for all experiments. Although the dynamic model offers higher accuracy under demanding driving, it requires the calibration of six parameters. In contrast, the simpler kinematic and axle models require calibration of only three parameters, which offers advantages in terms of computational efficiency and a lower risk of overfitting parameters to calibration data.

The second research question investigated how the wheel sensor-based DR method performed compared to standalone GNSS positioning. The results showed that for short periods of time, motion models can provide more accurate positioning compared to GNSS based positioning. In all the evaluated experiments, motion models initially achieved lower RMSE and absolute position error. However, as expected, the motion models eventually started to drift further from the RTK reference. This was most likely due measurement errors, measurement noise and model errors or uncertainties. GNSS generally provided more reliable and accurate positing after approximately 10-20 seconds. This shows that sensor based DR alone is not suitable as a positioning solution, but can provide short-term value during temporary GNSS signal degradation or outage.

The third research question investigated if sensor fusion of GNSS and sensor based motion models using an EKF could improve vehicle positioning compared to only using a wheel's sensor data. The results showed that the EKF consistently provided more accurate positioning compared to the DR baseline. In the turning experiment, the EKF gave lower RMSE and EPE compared to the motion model alone. This proves that combining GNSS data with vehicle sensor data can improve positioning estimates and reduce the accumulated drift over time. The lateral movement experiment showed that the EKF was capable of capturing lateral vehicle motion with relative accuracy. Although positing errors did accumulate over time leading to a few meters drift. Generally, the EKF gave a more robust and accurate trajectory estimation compared to DR.

The results suggest that wheel sensor-based dead reckoning can be combined with GNSS through sensor fusion to enhance positioning accuracy, or used as a short-term complementary positioning solution during GNSS outages. Future work could investigate whether the motion models can be combined or further developed. Since each model performs accurately in different driving conditions and velocities, the model selection could be adapted based on driving conditions. All models estimate x - and y -positions and the orientation θ , and this could be used to combine the models. Furthermore, the EKF could be extended to estimate additional vehicle parameters. WSS and SAS measurements likely contain biases that result in errors accumulating over time. If these biases could be modeled using the EKF, these errors could be reduced, resulting in more robust positioning accuracy.

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